# Transmission Capacity of Wireless Ad Hoc Networks: Successive Interference Cancellation vs. Joint Detection

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***Abstract*—The performance benefits of two interference cancellation methods, successive interference cancellation (SIC) and joint detection (JD), in wireless ad hoc networks are compared within the transmission capacity framework. SIC involves successively decoding and subtracting out strong interfering signals until the desired signal can be decoded, while higher-complexity JD refers to simultaneously decoding the desired signal and the signals of a few strong interferers. Tools from stochastic geometry are used to develop bounds on the outage probability as a function of the spatial density of interferers. These bounds show that SIC performs nearly as well as JD when the signal-to-interference ratio (SIR) threshold is less than one, but that SIC is essentially useless for SIR thresholds larger than one whereas JD provides a significant outage benefit regardless of the SIR threshold.**

## I. Introduction

Multi-user interference is perhaps the most serious impediment to successful communication in ad hoc networks. If no interference mitigation is performed, communications fail whenever an interfering transmitter is too close to a receiver who is attempting to decode a different transmission. As the density of simultaneous transmissions increases, it becomes more and more likely that interference prevents successful transmissions.

One possible remedy is to decode and cancel out the signals of sufficiently strong interferers, thereby reducing the effective interference level when trying to decode the intended signal. In this work, we quantify the benefit of two different interference cancellation methods, successive interference cancellation (SIC) and joint detection (JD), in the transmission capacity (TC) framework [1]. In the TC framework, a TX-RX pair separated by $d$ meters is surrounded by a Poisson field of interferers with spatial density $\lambda$ interferers/m$^2$. A communication is successful if the signal-to-interference ratio (SIR) is above a pre-defined threshold $\beta$. The relevant metric is the success probability of communications, and this quantity is clearly a decreasing function of the interferer density $\lambda$.

Efficient interference cancellation methods are expected to increase the communication success probability (for fixed density $\lambda$), and the objective of this paper is to quantify this increase for SIC and JC. This work is strongly motivated by [2], where interference cancellation was first investigated within the TC framework. In that work no specific cancellation technique is considered, but rather the simplifying assumption is made that all interferers within a specific radius of the receiver are cancellable. Although this allows for tractability, this simplifying model does not necessarily match the performance of actual IC methods. In this work, we study the precise conditions for two cancellation methods, SIC and JD, and develop outage probability lower bounds (i.e., success probability upper bounds) that allow for quantification of the benefit of the two techniques. We study the following two cancellation methods:

- **Successive Interference Cancellation** (SIC) decodes the signals from the strongest interferers and subtracts the re-encoded signal, effectively increasing SIR. This is iteratively repeated for up to the the first $k$ interferers, yielding a process called $k$-SIC. The successes of these steps are contingent on the information transmitted between users being sent at a rate within single-user channel capacity, i.e. SIR $\geq \beta$.
- **Joint Detection** (JD) uses a hybrid of SIC (superposition coding) and rate-splitting or time-sharing coding[4] to achieve all rates $(R_0, ..., R_k)$ within the $k+1$-user MAC capacity region $C_{\text{MAC}}$. The IC group size $k$ (number of interferers being jointly decoded with the intended transmission) may be increased successively, resulting in the process called $k$-JD.

By considering just the dominant (nearest) interferers and using some simplified necessary conditions for success, closed form outage probability lower bounds are derived. These lower bounds are then compared to yield a few interesting results:

- When the threshold SIR ($\beta$) is less than unity ($\beta < 1$), the outage bounds for SIC and JD are comparable to unconditional SIC (described later, the best imaginable IC scheme). That is, these realizable IC methods are very effective and almost always allow the first $k$ interferers to be effectively cancelled, making the $(k+1)^{\text{st}}$ interferer dominant. The outage probability for $k$-SIC and $k$-JD in these cases is $P_{\text{out}} = O(\lambda^{k+1})$. Another way to consider this is that for $\beta < 1$ if interferers are *strong enough to* *cause an outage*, then they are almost always *able to be cancelled* through these methods.

- When $\beta \geq 1$, the outage probability for SIC is only marginably better than without any IC methods, $P_{\text{out}}^{\text{SIC}} = O(\lambda)$. However, in this regime, JD still *performs well* especially in sparse networks; $P_{\text{out}}^{\text{JD}} = O(\lambda^{k+1})$.

## II. Preliminaries

### A. Network Model

We consider an infinite field of transmitters distributed according to a homogeneous 2-D Poisson Point process with density $\lambda$ m$^{-2}$. Each transmitter is associated with a receiver that is located a fixed distance $d$ meters away (the receivers are not a part of the PPP). Due to basic properties of the PPP, we may condition on a single receiver being located at the origin, denoted RX0, with its transmitter TX0 being located at a distance $d$. From the perspective of RX0, the locations of the interferers (TX1, TX2, ..) form a PPP with intensity $\lambda$ (by Slivnyaks Theorem [5], the distribution of TX1, TX2,.. is unaffected by conditioning on the location of the intended transmitter TX0).

The interferers are ordered by their distance to the origin (RX0), with distances denoted $X_1, X_2, ...$ . A useful property for this work is the fact that the square distances $X_1^2, X_2^2, ...$ form a 1-D PPP with intensity $\pi\lambda$ [3]. The distance to the intended transmitter is denoted $X_0 = d$.

We consider a path-loss only model with exponent $\alpha > 2$ (i.e., no fading). If we denote the signal transmitted by TX $i$ by $u_i$, the received signal at RX 0 is given by:

$$Y_0 = d^{-\alpha} u_0 + \sum_{i=1}^{\infty} X_i^{-\alpha} u_i.$$

To focus on the effect of multi-user interference, thermal noise is ignored.

We assume that all users transmit at rate equal to $R = \log_2(1+\beta)$. If no interference cancellation is performed, a communication is successful if and only if the received SIR $\frac{d^{-\alpha}}{\sum_{i=1}^{\infty} X_i^{-\alpha}}$ is larger than $\beta$. With SIC and JD, however, the conditions for successful communication are relaxed and are described as follows.

### B. Success conditions for SIC

We define $I_k$ as the sum of the interference from TX$k$ onward:

$$I_k = \sum_{j=k}^{\infty} X_j^{-\alpha}. \tag{1}$$

The successful transmission event without any IC methods can be written as

$$E_0 : \left( \frac{d^{-\alpha}}{I_1} \geq \beta \right). \tag{2}$$

The first step of $k = 1$ SIC is successful if the first interferer is decodable and subsequently the intended transmitter is decodable:

$$E_1 : \left( \frac{X_1^{-\alpha}}{d^{-\alpha} + I_2} \geq \beta \right) \cap \left( \frac{d^{-\alpha}}{I_2} \geq \beta \right). \tag{3}$$

Similarly, define the success event $E_k$ for the $k^{th}$ step of SIC as:

$$E_k : \left( \bigcap_{j=1}^{k} \frac{X_j^{-\alpha}}{d^{-\alpha} + I_{j+1}} \geq \beta \right) \cap \left( \frac{d^{-\alpha}}{I_{k+1}} \geq \beta \right). \tag{4}$$

Thus the overall success of $k$-SIC is the success of any of the steps $1, ..., k$:

$$P_{\text{succ}}^{k \text{ SIC}} = \bigcup_{j=0}^{k} E_j. \tag{5}$$

Also considered is Unconditional SIC, a.k.a. "For-Free" or FF-IC, for which interferers are considered to be cancelled "at-will." Note that FF-IC is the best imaginable IC scheme but is impossible to actually implement, so it will be useful for comparison only. The success of this algorithm is:

$$P_{\text{succ}}^{k \text{ FF}} = \left( \frac{d^{-\alpha}}{I_{k+1}} \geq \beta \right). \tag{6}$$

### C. Success conditions for JD

The success of joint detection is determined by considering the multiple-access capacity region from the $k$ nearest interferers and the desired transmitter to the reference receiver while treating the signals from interferers $k+1$ onwards as noise (i.e. the thermal noise is effectively $I_{k+1}$). Because all nodes transmit at rate $R$, the relevant capcity region is given by [4]:

$$C_{\text{MAC}} = \Bigg\{ (R_0, .., R_k) : R_0 = R_1 = ... = R_k = R,$$
$$|S| \cdot R \leq \log_2 \left( 1 + \frac{\sum_{j \in S} X_j^{-\alpha}}{I_{k+1}} \right) \forall S \subset \{0, 1, ..., k\} \Bigg\},$$

and success occurs if the vector $(R_0, ..., R_k) = (R, .., R)$ falls inside of this region. Note that the conditions for $k$-SIC success can be interpreted as requiring $(R, ..., R)$ to fall in a region strictly smaller than $C_{\text{MAC}}$.

## III. Outage Lower Bounds

### A. No IC

Note from [1] that the closest interferer dominates the outage probability which allows the outage probability to be bounded as follows:

*Lemma 3.1:* The outage probability with no IC methods is lower bounded as $P_{\text{out}}^{\text{No-IC}} \geq P_{\text{out,LB}}^{\text{No-IC}}$ where

$$\begin{aligned} P_{\text{out,LB}}^{\text{No-IC}} &= 1 - e^{-\lambda \pi d^2 \beta^{\frac{2}{\alpha}}} \\ &= \lambda \pi d^2 \beta^{\frac{2}{\alpha}} + O(\lambda^2). \end{aligned} \tag{7}$$

Therefore, by taking the inverse of this function, the contention density for a desired outage probability is upper bounded as:

$$\lambda(\epsilon) \leq \lambda_{\text{UB}}(\epsilon) = \frac{\epsilon}{\pi d^2 \beta^{\frac{2}{\alpha}}}. \tag{8}$$

### B. Unconditional SIC (FF-IC)

We make the following conclusion about the outage probability:

*Lemma 3.2:* The outage probability for $k$-level unconditional "for-free" SIC is lower bounded as $P_{\text{out}}^{k\ \text{FF}}(\lambda) \geq P_{\text{out, LB}}^{k\ \text{FF}}(\lambda)$, where

$$\begin{aligned} P_{\text{out, LB}}^{k\ \text{FF}}(\lambda) &= 1 - \left(\sum_{n=0}^{k} \frac{1}{n!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^n\right) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} \\ &= \frac{(\lambda\pi d^2\beta^{\frac{2}{\alpha}})^{(k+1)}}{(k+1)!} + O(\lambda^{k+2}). \end{aligned} \tag{9}$$

The proof is accomplished by making similar reasoning as the no-IC case. A sufficient condition for outage is when the $(k+1)^{\text{st}}$ interferer is too near relative to the intended transmitter i.e. $X_{k+1} \leq d^2\beta^{\frac{2}{\alpha}}$ where $X_{k+1} \sim$ Chi-Square with $2(k+1)$ degrees of freedom. This gives the TC [1] upper bound

$$\begin{aligned} c^{\text{FF}}(\epsilon) &\leq c_{\text{UB}}^{\text{FF}}(\epsilon) = (1-\epsilon)\cdot\lambda_{\text{UB}}^{\text{FF}}(\epsilon), \\ \lambda_{\text{UB}}^{\text{FF}}(\epsilon) &= \frac{\sqrt[k+1]{\epsilon(k+1)!}}{\pi d^2 \beta^{\frac{2}{\alpha}}}. \end{aligned} \tag{10}$$

### C. Successive Interference Cancellation

*1) SIC $k=1$:*

*Lemma 3.3:* For $k=1$ SIC, the outage probability is lower bounded as $P_{\text{out}}^{k=1\ \text{SIC}}(\lambda) \geq P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda)$, where

$$\begin{aligned} &P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda) \\ &= \begin{cases} 1-(d^2\beta^{\frac{2}{\alpha}}\lambda\pi+1)e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} & \text{if } \beta < 1 \\ 1-(d^2\beta^{\frac{-2}{\alpha}}\lambda\pi+1)e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} & \text{if } \beta \geq 1. \end{cases} \end{aligned} \tag{11}$$

*Proof:* Define the set of square interferer distances which satisfy the conditions of success in terms of only the first 2 interferers as $J_1$. It is clear this already provides an upper bound on outage probability as the number of interfering users is reduced.

Now define the set $I_1$ for which $(x_1^2, x_2^2) \in I_1 \Rightarrow (x_1^2, x_2^2) \in J_1$. The conditions on set $I_1$ are relaxed from those of the conditions of set $J_1$ but as we will see, allow a closed form.

$$\begin{aligned} \underline{\beta < 1}:\ I_1^{(1)} &= \{(x_1^2, x_2^2) : x_2^2 \geq d^2\beta^{\frac{2}{\alpha}}, x_1^2 < d^2\beta^{\frac{2}{\alpha}}\} \\ &\quad \cap \{(x_1^2, x_2^2) : x_2^2 \geq x_1^2, x_1^2 \geq d^2\beta^{\frac{2}{\alpha}}\} \end{aligned}$$

$$\begin{aligned} \underline{\beta \geq 1}:\ I_2^{(1)} &= \{(x_1^2, x_2^2) : x_2^2 \geq d^2\beta^{\frac{2}{\alpha}}, x_1^2 < d^2\beta^{\frac{-2}{\alpha}}\} \\ &\quad \cap \{(x_1^2, x_2^2) : x_2^2 \geq x_1^2, x_1^2 \geq d^2\beta^{\frac{2}{\alpha}}\} \end{aligned}$$

Note that conditions for $I_2^{(1)}$ indicate a region of uncancellability when $\beta \geq 1$, illustrated in Fig. 1.

We write the following bounds for the success and outage probability:

Case 1. When $\beta < 1$:

$$\begin{aligned} P_{\text{succ}}^{k=1\ \text{SIC}}(\lambda) &\leq P_{\text{succ,UB}}^{k=1\ \text{SIC}}(\lambda) \\ P_{\text{succ,UB}}^{k=1\ \text{SIC}}(\lambda) &= \int\int_{I_1^{(1)}} f_{X_1^2, X_2^2}(x_1^2, x_2^2)\, dx_2^2\, dx_1^2 \\ &= (d^2\beta^{\frac{2}{\alpha}}\lambda\pi + 1)e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} \end{aligned}$$

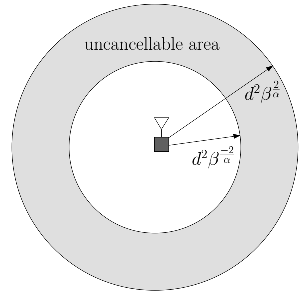


Fig. 1. Uncancellable region for SIC in terms of distance to $RX0$. Interferers in this region have a large likelihood of being undecodable and uncancellable. Note the difference between the bounds is the change in sign of the exponent on $\beta$. Thus, this region disappears when $\beta < 1$.

$$\begin{aligned} P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda) &= 1 - P_{\text{succ,UB}}^{k=1\ \text{SIC}}(\lambda) \\ &= 1-(d^2\beta^{\frac{2}{\alpha}}\lambda\pi+1)e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} \qquad (12) \\ &= \frac{(\lambda\pi d^2\beta^{\frac{2}{\alpha}})^2}{2!} + O(\lambda^3) \qquad (13) \end{aligned}$$

Due to the convex increasing nature of the outage probability with respect to $\lambda$, when $\beta < 1$:

$$P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda) \geq \frac{(\lambda\pi d^2\beta^{\frac{2}{\alpha}})^2}{2!}.$$

Case 2. When $\beta \geq 1$:

$$\begin{aligned} P_{\text{succ}}^{k=1\ \text{SIC}}(\lambda) &\leq P_{\text{succ,UB}}^{k=1\ \text{SIC}}(\lambda) \\ P_{\text{succ,UB}}^{k=1\ \text{SIC}}(\lambda) &= \int\int_{I_2^{(1)}} f_{X_1^2, X_2^2}(x_1^2, x_2^2)\, dx_2^2\, dx_1^2 \\ &= \left[(d^2\beta^{\frac{-2}{\alpha}}\lambda\pi)+1\right]e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} \end{aligned}$$

$$\begin{aligned} P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda) &= 1-(d^2\beta^{\frac{-2}{\alpha}}\lambda\pi+1)e^{-\lambda\pi d^2\beta^{\frac{2}{\alpha}}} \qquad (14) \\ &= \lambda\pi d^2(\beta^{\frac{2}{\alpha}} - \beta^{\frac{-2}{\alpha}}) + O(\lambda^2) \qquad (15) \end{aligned}$$

Since the lower bound on outage probability is convex increasing in $\lambda$, we may lower bound it by its dominant term.

$$P_{\text{out,LB}}^{k=1\ \text{SIC}}(\lambda) \geq \lambda\pi d^2(\beta^{\frac{2}{\alpha}} - \beta^{\frac{-2}{\alpha}})$$

By combining eqns. (12) and (14) note that we have yielded the result. ■

Solving the dominant term bound for small outage probability $\epsilon$ allows the upperbound function on contention density and TC for $k=1$ SIC:

$$c^{\text{SIC}} \leq c_{\text{UB}}^{\text{SIC}}(\epsilon) = (1-\epsilon)\cdot\lambda_{\text{UB}}^{\text{SIC}}(\epsilon)$$

$$\lambda_{\text{UB}}^{k=1\ \text{SIC}}(\epsilon) = \begin{cases} \frac{\epsilon}{\pi d^2(\beta^{\frac{2}{\alpha}} - \beta^{\frac{-2}{\alpha}})} & \text{if } \beta \geq 1 \\ \frac{\sqrt{2\epsilon}}{\pi d^2\beta^{\frac{2}{\alpha}}} & \text{if } \beta < 1. \end{cases}$$

*2) SIC $k \geq 2$:* By again considering the strongest $k+1$ interferers and itemizing conditions for their success and integrating PDF's , we obtain the following theorem.

*Theorem 3.4:* For $k \geq 2$ SIC, the outage probability is lower bounded as $P_{\text{out}}^{k\text{ SIC}}(\lambda) \geq P_{\text{out,LB}}^{k\text{ SIC}}(\lambda)$, where

$$P_{\text{out,LB}}^{k\text{ SIC}}(\lambda) = \begin{cases} 1 - \left(\sum_{n=0}^{k} \frac{1}{n!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^n\right) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} & \text{if } \beta < 1 \\ 1 - (d^2 \beta^{\frac{-2}{\alpha}} \lambda\pi + 1) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} & \text{if } \beta \geq 1. \end{cases} \quad (16)$$

As a result of this theorem, taking a Taylor's series expansion about small contention density, and inverting results in the contention density upper bound:

$$\lambda_{\text{UB}}^{k\text{ SIC}}(\epsilon) = \begin{cases} \frac{\sqrt[(k+1)]{(k+1)!\epsilon}}{\pi d^2 \beta^{\frac{2}{\alpha}}} & \text{if } \beta < 1 \\ \frac{\epsilon}{\pi d^2 (\beta^{\frac{2}{\alpha}} - \beta^{-\frac{2}{\alpha}})} & \text{if } \beta \geq 1. \end{cases} \quad (17)$$

## D. Joint Detection

*1) JD $k = 1$:* Consider the joint decoding of the intended transmitter and the strongest interferer. We show the lower bounds are on the order of the FF-IC bounds.

*Lemma 3.5:* The outage probability for $k = 1$ Joint Detection is lower bounded as $P_{out}^{k=1\text{ JD}}(\lambda) \geq P_{out,LB}^{k=1\text{ JD}}(\lambda)$, where

$$\begin{aligned} P_{out,LB}^{k=1\text{ JD}}(\lambda) &= 1 - (\lambda\pi d^2 \beta^{\frac{2}{\alpha}} + 1) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} \\ &= \frac{1}{2!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^2 + O(\lambda^3). \end{aligned}$$

*Proof:* The condition on success of the $k = 1$ joint detection procedure is:

$$((R_0) = (R) \in C_{\text{MAC}}) \cup ((R_0, R_1) = (R, R) \in C_{\text{MAC}}) . \quad (18)$$

Define $J_1$ as the set of interferer distances satisfying the above conditions in terms of only the first two interferers. We now define the set $I_1$ which is superset of $J_1$, $J_1 \subseteq I_1$. It will be shown that $I_1$ is also a reasonable approximation for $J_1$.

$$\begin{aligned} I_1 &= \{(x_1^2, x_2^2) : x_2^2 \geq d^2 \beta^{\frac{2}{\alpha}}, x_1^2 < d^2 \beta^{\frac{2}{\alpha}}, x_2^2 \geq x_1^2 \geq 0\} \\ &\cap \{(x_1^2, x_2^2) : x_1^2 \geq d^2 \beta^{\frac{2}{\alpha}}, x_2^2 \geq x_1^2 \geq 0\} \end{aligned}$$

$$\begin{aligned} P_{succ}^{k=1\text{ JD}}(\lambda) &\leq P_{succ,UB}^{k=1\text{ JD}}(\lambda) \\ P_{succ,UB}^{k=1\text{ JD}}(\lambda) &= \int\int_{I_1} f_{X_1^2, X_2^2}(x_1^2, x_2^2)\, dx_2^2\, dx_1^2 \\ &= (d^2 \beta^{\frac{2}{\alpha}} \lambda\pi + 1) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} \end{aligned}$$

$$\begin{aligned} P_{out,LB}^{k=1\text{ JD}}(\lambda) &= 1 - P_{succ,UB}^{k=1\text{ JD}}(\lambda) \\ &= 1 - (d^2 \beta^{\frac{2}{\alpha}} \lambda\pi + 1) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} \quad (19) \\ &= \frac{1}{2!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^2 + O(\lambda^3) \quad (20) \end{aligned}$$

Since the outage probability is convex increasing, the dominant term in the Taylor series will be a lower bound itself.

$$P_{out,LB}^{k=1\text{ JD}}(\lambda) \geq \frac{1}{2!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^2 \quad (21)$$

■

This outage probability is lower (better) than that of SIC for $\beta \geq 1$ and is on the order of FF-IC. The implication being that the main limitation of SIC is the uncancellable annulus.

*2) JD $k \geq 2$:* By using the observation that for JD most outages are caused by a "near-field event" from the $(k+1)^{st}$ interferer (i.e. that interferer is capable of causing outage itself), we derive a lower bound on the outage probability as we did in the FF-IC case.

*Theorem 3.6:* The outage probability for $k$-JD is bounded as $P_{\text{out}}^{k\text{ JD}}(\lambda) \geq P_{\text{out, LB}}^{k\text{ JD}}(\lambda)$, where

$$\begin{aligned} P_{\text{out, LB}}^{k\text{ JD}}(\lambda) &= 1 - \left(\sum_{n=0}^{k} \frac{1}{n!}(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^n\right) e^{-\lambda\pi d^2 \beta^{\frac{2}{\alpha}}} \\ &= \frac{(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^{(k+1)}}{(k+1)!} + O(\lambda^{k+2}). \quad (22) \end{aligned}$$

As a consequence,

$$P_{\text{out, LB}}^{k\text{-JD}}(\lambda) \geq \frac{(\lambda\pi d^2 \beta^{\frac{2}{\alpha}})^{(k+1)}}{(k+1)!} \quad (23)$$

$$\lambda_{\text{UB}}^{k\text{-JD}}(\epsilon) = \frac{\sqrt[k+1]{(k+1)!\epsilon}}{\pi d^2 \beta^{\frac{2}{\alpha}}}. \quad (24)$$

# IV. Numerical Results

To simulate the network, a Matlab program was used which realized a subset of a 1-D PPP, which corresponds to a subset of a 2-D PPP as noted in the Preliminaries. The square distances to the closest $100$ interferers are realized. Then, using channel properties indicated in figures, SIR and channel capacity are calculated and compared to determine success or outage of transmission over several thousand iterations to realize a stable probability approximation.

Figures 2 and 3 show output plots of outage probability against contention density for two different values of threshold SIR $\beta$ less than and greater than unity. Also included are the derived lower bounds which are shown with a dashed line and markers which correspond to their respective IC methods. Note that while the SIC bound changes against $\beta$, the JD lower bound is always the same as the FF bound. These plots support earlier conclusions that SIC and JD have substantial benefits on the order of unconditional SIC when threshold SIR is less than unity. However, SIC performs only marginally better than without any IC when $\beta \geq 1$, but JD still performs very well.

Figure 4 shows the output plot of simulations of Outage probability vs. $\beta$ which illustrates the discontinuity in the outage behavior of SIC above and below $\beta = 1$. This also shows that the bounds are good approximations for SIC across all values of $\beta$.

Figure 5 illustrates the behavior of the IC methods JD and SIC against group size $k$. Note that as group size increases beyond about 3, the outage probabilities for JD and SIC achieve a smaller gain for each consecutive increase in $k$ while the toy FF-IC outage can be arbitrarily small. This behavior is not captured in the derived lower bounds.

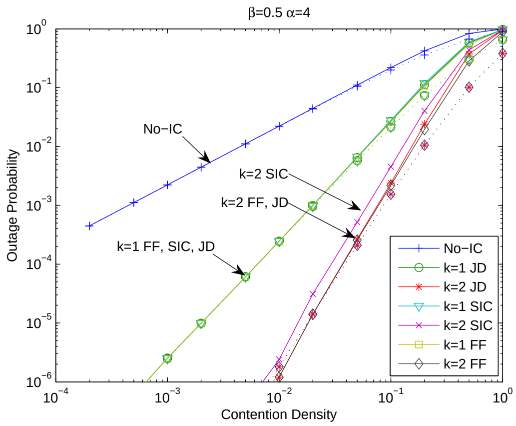


Fig. 2. Simulated Outage Probability vs. Contention Density ($\lambda$) for JD, SIC $k = 1, 2$ ($\beta = 0.5$). Dashed line indicates corresponding lower bound outage probability. Note that SIC and JD are on the same order as Uncond. SIC (FF) and large performance gains for increased $k$.

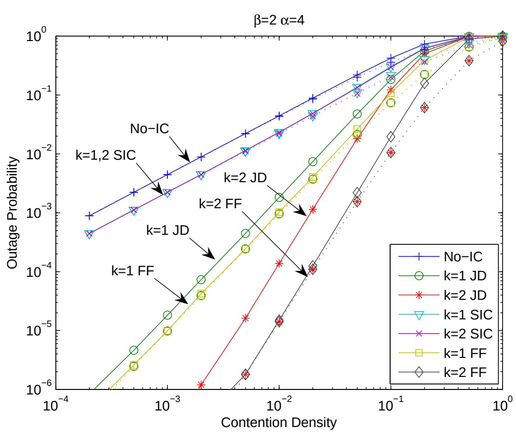


Fig. 3. Simulated Outage Probability vs. Contention Density ($\lambda$) for JD, SIC $k = 1, 2$ ($\beta = 2$). Dashed line indicates corresponding lower bound outage probability. Note the SIC bounds are on order with No-IC while JD still has a large benefit.

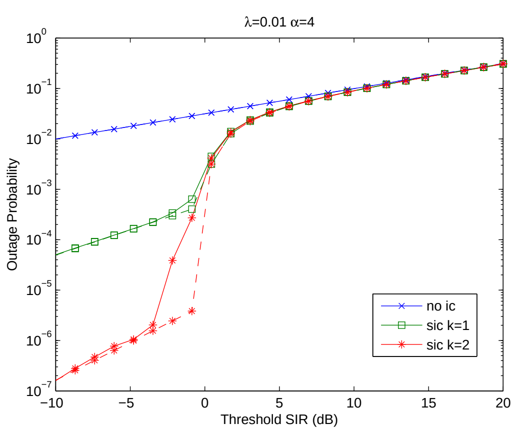


Fig. 4. Simulated Outage Probability vs. $\beta$ for SIC $k = 1, 2$ ($\lambda = 0.01, \alpha = 4.0$). Dashed line indicates lower bound. Note the strongly disjoint behavior against $\beta$ around 1 which is captured well in the derived lower bound.

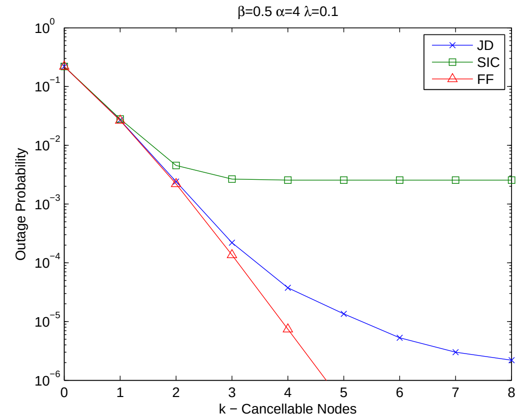


Fig. 5. Simulated Outage Probability vs. IC group size $k$ ($\lambda = 0.1, \beta = 0.5, \alpha = 4.0$). Note that JD performs about as well as Uncond. SIC (FF) up to $k = 3$ and SIC no longer has performance gains with $k \geq 3$

## V. Conclusion

Lower bounds for outage probability are derived for joint detection and successive interference cancellation of varying group size $k$. Without any IC methods, the outage probability is $O(\lambda)$. For $k$-JD, this outage probability is $O(\lambda^{k+1})$ while for SIC $P_{\text{out}}$ is $O(\lambda^{k+1})$ for SIR $\beta < 1$ and $O(\lambda)$ for $\beta \geq 1$. The main inhibitor for SIC performance is found to be an 'uncancellable' annulus in space relative to the intended transmitter where the intended transmission blocks the interferers from being decoded and cancelled. Simulations show the lower bounds are tight in a majority of cases. The benefits of IC are seen to diminish for $k \geq 3$ . The benefits of JD are tremendous and surpass those of SIC but depend largely on good code schemes, near perfect channel state information and more complicated hardware than is required for SIC.

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