Optimal Amplify and Forward Strategy for Two-Way Relay Channel with Multiple Relays

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Abstract— **An iterative algorithm is proposed to achieve the optimal rate region in a two-way relay channel, where two nodes want to exchange data with each other using multiple relays, and each relay employs an amplify and forward strategy. The iterative algorithm solves a power minimization problem at every step, subject to minimum signal-to-interference-and-noise ratio constraints, which is non-convex, however, for which the Karush Kuhn Tucker conditions are sufficient for optimality. Using simulations, the achievable rate region of the iterative algorithm is compared with the cut-set upper bound; the gap is shown to be quite small for most cases.**

I. INTRODUCTION

In a two-way relay channel, two nodes T_1 and T_2 want to exchange information with each other with the help of one or more relay nodes. The two-way relay channel models the communication scenario where the destination terminal also has some data to send to source terminal e.g. downlink and uplink in cellular communication, or packet acknowledgments in a wireless network. The general discrete memoryless two-way relay channel was introduced in [1], and the multiple antenna two-way relay channel in [2]. In the literature, the two-way relay channel is also known by several other names, including the bidirectional relay channel [3]–[5] and analog network coding [6]. The key idea with the two-way relay channel is that each terminal can cancel the interference (generated by its own transmission) from the signal it receives from the relay to recover the transmission from the other terminal. The idea is reminiscent of work in network coding [7], though note that here the coding is done in the analog domain, [6] rather than in digital domain [7].

Finding the optimal transmission strategy (capacity region) for the two-way relay with a single relay node has lately attracted a lot of attention [3]–[5], [8]–[13]. Achievable sum rate expressions (sum of the rates achievable from $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ links) have been derived using amplify and forward (AF), decode and forward (DF) and compress and forward (CF) at the relay for the half-duplex two-way relay channel in [2] and [3]–[5], and for the general discrete memoryless two-way channel [8]. It is shown that in a two-way relay channel, it is possible to remove the $\frac{1}{2}$ rate loss factor in spectral efficiency due to the half duplex assumption on the nodes. More recently, achievable rate regions have been derived using the deterministic channel approach in [9], and by choosing a suitable relay mapping function, together with LDPC codes in [12], [13]. For the AWGN two-way relay channel (no fading), achievable rate regions have been derived, using nested lattice coding and DF at the relay in [10], [11]. The achievable rate region [3]–[5], [8]–[13] does not meet the upper bound [14], in general, and consequently, the problem of finding the capacity region of the two-way relay channel is currently open.

The problem of finding the capacity region of the two-way relay channel becomes even more challenging when there are multiple relay nodes that can help T_1 and T_2. The problem becomes hard, because it is known that for the one-way relay channel (no communication from T_2 to T_1) with multiple relay nodes, DF does not work well [15], while the partial DF and distributed CF [15] lead to complicated achievable rate regions that are very hard to compute and compare with an upper bound. The same conclusion holds true for the two-way relay channel; the only simple strategy that is well suited for multiple relay nodes is AF. With this motivation, in this paper we attempt to find the optimal relay beamformers that maximize the achievable rate region of the two-way relay channel with AF. For the one-way relay channel with multiple relays, optimal relay beamformers have been found in [16].

We solve the problem of finding optimal relay beamformers in a two-way relay channel with multiple relay nodes by recasting it as an iterative power minimization algorithm. The proposed iterative algorithm, at each step, solves a power minimization problem with minimum signal-to-interference-noise (SINR) constraints, for which satisfying the Karush Kuhn Tucker (KKT) conditions [17], [18] are sufficient for optimality. We consider both the sum power constraint across relays, as well as an individual relay power constraint. Using simulations we show that the gap between the achievable rate region of the proposed iterative algorithm and a cut-set upper bound [21] is quite small.

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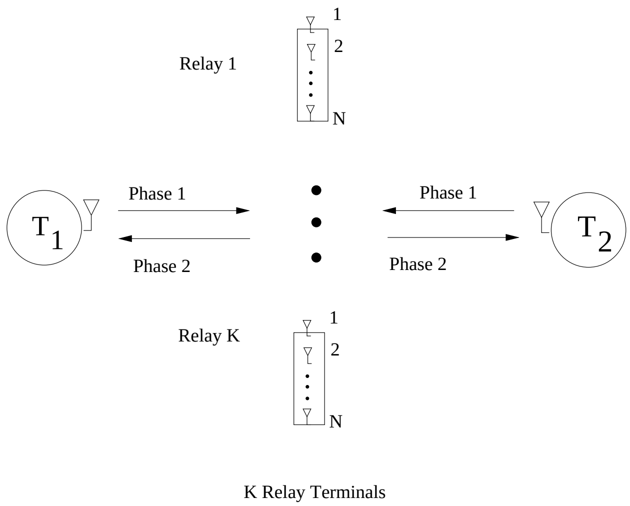

Fig. 1. Two-way relay channel system model with two phase communication

Notation: The following notation is used in this paper. The superscripts * represent the transpose conjugate. $\mathbb{C}$ denotes the field of complex numbers. $\mathbf{m}$ denotes a vector and m_i the i^{th} element of $\mathbf{m}$. $||\cdot||$ denotes the usual Euclidean norm of a vector and $|\cdot|$ denotes the absolute value of a scalar. $\mathbf{I}_m$ is a $m \times m$ identity matrix. $x \sim \mathcal{CN}(0, \sigma)$ means x is a circularly symmetric complex Gaussian random variable with zero mean and variance σ.

II. System Model

We consider a wireless network where there are two terminals T_1 and T_2 who want to exchange information via K relays, as shown in Fig. 1. The K relays do not have any data of their own and only help T_1 and T_2 communicate. The K relays are assumed to be located randomly and independently so that the channel coefficients between each relay and T_1 and T_2 are independent. We also assume that there is no direct path between T_1 and T_2 and that they can communicate only through the K relays. This is a realistic assumption when relaying is used for coverage improvement in cellular systems, since at the cell edge the signal to noise ratio is extremely low for the direct path. In ad-hoc networks, this is the case when two terminals want to communicate, but are out of each other's transmission range.

We assume that both the terminals T_1 and T_2 each have a single antenna, while all the K relays have N antennas each. We further assume that both the terminals and all the relays can operate only in half-duplex mode (cannot transmit and receive at the same time). The communication protocol is summarized as follows [2]. In any given time slot, for the first half of a time slot, called the *transmit phase*, both T_1 and T_2 are scheduled to transmit and all the relays receive a superposition of the signals transmitted from T_1 and T_2. In the second half of the time slot, called the *receive phase*, all the relays are scheduled to transmit simultaneously and both the terminals receive. Both T_1 and T_2 are assumed to have power constraint of P, while for relays we assume two different power constraints, the sum power constraint where the sum of the power of all relays is $\leq P_R$ or the individual power constraint where each relay has power constraint of P_R.

III. Channel and Signal Model

Throughout this paper we assume that all the channels are frequency flat slow fading block fading channels. In a block of time duration T_c (called the coherence time), the channel coefficients remain constant and change independently from block to block. Let the forward channel between T_1 and the k^{th} relay be $\mathbf{h}_k$ and the backward channel between k^{th} relay and T_1 be $\mathbf{h}^r_k$. Similarly let the forward channel between k^{th} relay and T_2 be $\mathbf{g}_k$ and the backward channel between T_2 and the k^{th} relay be $\mathbf{g}^r_k$. We assume that $\mathbf{h}_k, \mathbf{g}^r_k \in \mathbb{C}^{N\times 1}, \mathbf{h}^r_k, \mathbf{g}_k \in \mathbb{C}^{1\times N}$ with independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ entries.

The signal model under consideration is as follows. The $N \times 1$ received signal at the k^{th} relay is given by

$$\mathbf{r}_k = \sqrt{\frac{P}{M}}\mathbf{h}_k x_1 + \sqrt{\frac{P}{M}}\mathbf{g}^r_k x_2 + \mathbf{n}_k \tag{1}$$

if x_1 and x_2 are the signals transmitted from T_1 and T_2 to be decoded at T_2 and T_1 respectively, with $\mathbb{E}\{{x_1}^* x_1\} = \mathbb{E}\{{x_2}^* x_2\} = 1$, P is the power transmitted by T_1 and T_2, respectively. The noise $\mathbf{n}_k$ is the $N \times 1$ spatio-temporal white complex Gaussian noise independent across relays with $\mathbb{E}(\mathbf{n}_k\mathbf{n}_k^*) = \mathbf{I}_N$. Relay k processes its incoming signal to transmit a $N \times 1$ signal $\mathbf{t}_k = \mathbf{W}_k\mathbf{r}_k$ with $\sum_{k=1}^K \mathbb{E}\{\mathbf{t}_k^*\mathbf{t}_k\} \leq P_R$ (sum power constraint) or $\mathbb{E}\{\mathbf{t}_k^*\mathbf{t}_k\} \leq P_R$ (individual power constraint) in the receive phase. The received signals y_1 and y_2 at terminal T_1 and T_2, respectively, are

$$y_1 = \sum_{k=1}^K \mathbf{h}^r_k\mathbf{t}_k + z_1, \quad y_2 = \sum_{k=1}^K \mathbf{g}_k\mathbf{t}_k + z_2, \tag{2}$$

where z_1 and z_2 are white complex Gaussian noise vectors with $\mathbb{E}(z_1 z_1^*) = \mathbb{E}(z_2 z_2^*) = 1$.

Throughout this paper we assume that both T_1 and T_2 perfectly know $\{\mathbf{h}_k, \mathbf{h}^r_k, \mathbf{g}_k, \mathbf{g}^r_k\}\ \forall\ k,\ k = 1, 2, , \dots K$ in the receive mode. To be precise, in the receive phase (i.e. when T_1 and T_2 receive signal from all the relays), T_1 and T_2 both know $\{\mathbf{h}_k, \mathbf{g}_k\}$ and $\{\mathbf{h}^r_k, \mathbf{g}^r_k\}\ \forall\ k,\ k = 1, 2, , \dots K$. We also assume that no transmit channel state information is available at T_1 and T_2, i.e. in the transmit phase T_1 and T_2 have no information about what the realization of $\mathbf{h}_k$ and $\mathbf{g}_k$ is going to be when it transmits its signal to all the relays in the transmit phase, respectively. We assume that each relay knows $\mathbf{h}_k, \mathbf{g}^r_k, \mathbf{g}_k, \mathbf{h}^r_k$ for all $k = 1, 2 \dots, K$.

IV. Optimal AF strategy for two-way relay channel

In this section we will find relay beamformers that maximize the achievable rate region of the two-way relay channel with AF. For simplicity of exposition, in this section we consider the case when each relay nodes has a single antenna, $N = 1$. Generalizations to $N > 1$ are straightforward, and will be described later.

Because of single antenna restriction on each relay node, the channel between T_1 and relay k is denoted by h_k and between relay k and T_2 denoted by g_k. For the reverse direction the channel coefficients are the same as in forward direction but with an added superscript r, e.g. channel coefficient between relay k and T_1 is denoted by h_k^r. With AF strategy, each relay node transmits the received signal multiplied with w_k to both T_1 and T_2. Thus, if x_1 and x_2 is the transmitted signal from T_1 and T_2, respectively, then the received signal at T_1, y_1, and T_2, y_2 is

$$\begin{aligned} y_1 &= \sum_{k=1}^{K} \sqrt{P} h_k^r w_k g_k^r x_2 + \sqrt{P} h_k^r w_k h_k x_1 + h_k^r w_k n_k + z_1, \\ y_2 &= \sum_{k=1}^{K} \sqrt{P} g_k w_k h_k x_1 + \sqrt{P} g_k w_k g_k^r x_1 + g_k w_k n_k \\ &\quad + z_2, \end{aligned} \tag{3}$$

where $n_k, \ \forall\ k = 1, \ldots, K$ is $\mathcal{CN}(0,1)$ noise added at relay k and z_1, and z_2 are $\mathcal{CN}(0,1)$ added at T_1 and T_2. Since x_1 and x_2 are known at T_1 and T_2, respectively, their contribution can be removed from the received signal at T_1 and T_2, respectively. Let the rate of transmission from T_1 to T_2 be R_{12} and from T_2 to T_1 be R_{21}, then from (3)

$$\begin{aligned} R_{12} &= \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} g_k w_k h_k\right|^2\right)}{1 + \sum_{k=1}^{K} |g_k w_k|^2}\right), \\ R_{21} &= \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} h_k^r w_k g_k^r\right|^2\right)}{1 + \sum_{k=1}^{K} |h_k^r w_k|^2}\right). \end{aligned}$$

Thus, the achievable rate region for the two-way relay channel with AF for a sum power constraint across all relays, i.e. $p_R = P\sum_{k=1}^{K}(|w_k h_k|^2 + |w_k g_k^r|^2) + \sum_{k=1}^{K} |w_k|^2 \leq P_R$ is the set $\mathcal{R}(P, P_R) = \cup_{p_R \leq P_R}(R_{12}, R_{21})$ and for individual power constraint at each relay, i.e. $p_{kR} = P(|w_k h_k|^2 + |w_k g_k^r|^2) + |w_k|^2 \leq P_R$ is the set $\mathcal{R}(P, P_R) = \cup_{p_{kR} \leq P_R,\ k=1,\ldots,K}(R_{12}, R_{21})$. Therefore, the problem is to find optimal w_k's that achieve the boundary points of the region $\mathcal{R}(P, P_R)$, for both the sum power constraint and an individual power constraint.

For the one-way relay channel, no communication from T_2 to T_1, optimal w_k's have been found in [16] to maximize R_{12}. The solution of [16], provides an upper bound on individual rates R_{12} and R_{21} for the two-way relay channel, and is equivalent to solutions where R_{12} or R_{21} is greedily maximized disregarding the other.

To find the optimal achievable rate region of the two-way relay channel with AF, we need to find w_k's such that $R_{sum} = R_{12} + R_{21}$ is maximized, for each $\beta \in [0,1]$, where $R_{12} = \beta R_{sum}$, and $R_{21} = (1-\beta) R_{sum}$ [19]. Essentially, we need to maximize the sum rate, under the constraint that the rate R_{12} is at least β times the sum rate and the rate R_{21} is at least $1-\beta$ times the sum rate, for each $\beta \in [0,1]$. Special case of $\beta = 0$ or $\beta = 1$ corresponds to corner points of the capacity region, where only R_{12} ($R_{21} = 0$) or R_{21} ($R_{12} = 0$) is maximized, while $\beta = \frac{1}{2}$ corresponds to the symmetric point on the capacity region where $R_{12} = R_{21}$. Next, we only consider the sum power constraint across the relays. For individual power constraints the same procedure can be applied as explained in footnote 1. Thus, the optimization problem can be formulated as follows.

$$\begin{aligned} \text{Max}_{w_k} \quad & R_{sum} \\ \text{subj. to} \quad & \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} g_k w_k h_k\right|^2\right)}{1+\sum_{k=1}^{K} |g_k w_k|^2}\right) \geq \beta R_{sum}, \\ & \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} h_k^r w_k g_k^r\right|^2\right)}{1+\sum_{k=1}^{K} |h_k^r w_k|^2}\right) \geq (1-\beta) R_{sum}, \\ & \textstyle\sum_{k=1}^{K} P(|w_k h_k|^2 + |w_k g_k^r|^2) + |w_k|^2 \leq P_R. \end{aligned} \tag{4}$$

Thus, we have to maximize R_{sum} subject to rate and power constraints. Alternatively, we could fix a particular value for R_{sum}, and find the minimum power required to achieve that value. If the minimum power required is less than the power constraint, then we can increase the value of R_{sum}, otherwise decrease the value of R_{sum}, and again find the minimum power required to achieve that rate. Continuing in this iterative fashion, we can converge on the optimal value of R_{sum} that satisfies the power constraint. Therefore an equivalent problem to this problem is the following iterative power minimization problem subject to rate constraints,

$$\begin{aligned} \text{Min}_{w_k} \quad & P\textstyle\sum_{k=1}^{K}(|w_k h_k|^2 + |w_k g_k^r|^2) + \sum_{k=1}^{K} |w_k|^2 \\ \text{subject to} \quad & \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} g_k w_k h_k\right|^2\right)}{1+\sum_{k=1}^{K} |g_k w_k|^2}\right) \geq \beta R_{sum}^u, \\ & \log\left(1 + \frac{P\left(\left|\sum_{k=1}^{K} h_k^r w_k g_k^r\right|^2\right)}{1+\sum_{k=1}^{K} |h_k^r w_k|^2}\right) \geq (1-\beta) R_{sum}^u, \end{aligned} \tag{5}$$

where at each iteration R_{sum}^u is changed to maximize the achievable rate, subject to power constraint. To be precise, let the value of R_{sum}^u at iteration i be $R_{sum}^{u,i}$, and the solution to (5) is feasible (i.e. if $p_R \leq P_R$) [1], then $R_{sum}^{u,i}$ is incremented in next iteration, otherwise decreased. Let the optimal value of R_{sum}^u be $R_{sum}^{u,opt}$. Then with this iterative algorithm, $|R_{sum}^{u,opt} - R_{sum}^{u,i+1}| < |R_{sum}^{u,opt} - R_{sum}^{u,i}|$ and $|R_{sum}^{u,opt} - R_{sum}^{u,i+1}| \to 0$ as $i \to \infty$. Thus, by using sufficient number of iterations, the output of the iterative algorithm can be made arbitrarily close to the optimal value. Choice of the step size of increase or decrease determines the speed of convergence to the optimal rate R_{sum}^u, for which $p_R \leq P_R$. One possible starting point for R_{sum}^u is 2 times the maximum R_{12} provided by [16] for one way relay channel. The step size can be chosen by bisection between the last feasible R_{sum}^u (initially 0) and the last infeasible R_{sum}^u. Even though this equivalent problem provides a solution to (4) in a iterative manner, the problem (5) is in general non-convex, and not easy to solve. To overcome this limitation, we recast the problem (5) as a standard power

[1]For an individual power constraint the same can be done by checking at each iteration whether the obtained solution p_R is feasible with individual power constraints or not.

$$\mathbf{A} = \begin{bmatrix} P(|h_1|^2+|g_1^r|^2)+1 & 0 & \dots & 0 \\ 0 & P(|h_2|^2+|g_2^r|^2)+1 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \dots & 0 & P(|h_K|^2+|g_K^r|^2)+1 \end{bmatrix},$$

minimization problem subject to signal-to-interference-noise ratio (SINR) [18], where the forwarded noise from each relay plays the role of interference. For a given β and R^u_{sum}, the problem (5) is of the form

$$\begin{array}{ll} \text{Min}_{w_k} & P\sum_{k=1}^K(|w_k h_k|^2+|w_k g_k^r|^2)+\sum_{k=1}^K|w_k|^2 \\ \text{subject to} & \frac{\left|\sum_{k=1}^K g_k w_k h_k\right|^2}{1+\sum_{k=1}^K|g_k w_k|^2} \geq \frac{2^{\beta R^u_{sum}}-1}{P} := \gamma_0 \\ & \frac{\left|\sum_{k=1}^K h_k^r w_k g_k^r\right|^2}{1+\sum_{k=1}^K|h_k^r w_k|^2} \geq \frac{2^{(1-\beta)R^u_{sum}}-1}{P} := \gamma_1. \end{array} \quad (6)$$

This problem again is non-convex, however, it is of the form

$$\begin{array}{ll} \text{Min} & f(x) \\ \text{subject to} & ||\mathbf{a}_i(x)||^2-|b_i(x)|^2 \leq 0,\ \forall\ i, \end{array} \quad (7)$$

where $f(x)$ is a convex function, $\mathbf{a}_i(x)$ is an affine function of x and $b_i(x) \geq 0 \quad \forall\ i$. Note that b_i, which is either $\sum_{k=1}^K g_k w_k h_k$ or $\sum_{k=1}^K h_k^r w_k g_k^r$, can be less than zero or complex for $i=1,2$, however, by scaling g_k's by appropriate phases, b_i can be made real and positive, without changing the objective function or the constraints [2].

For the problem (7), it has been shown in [18], that if the problem is strictly feasible, then KKT conditions [17] are necessary and sufficient to find the optimal solution. Note that for any value of R^u_{sum}, there exist w_k's that strictly satisfy the rate constraint, therefore, the problem (6) is strictly feasible and consequently the KKT conditions are sufficient for optimality. The Lagrangian of problem (6) is of the form

$$\begin{aligned} \mathcal{L} &= \mathbf{w}\mathbf{A}\mathbf{w}^* + \lambda_1\left(\mathbf{w}\mathbf{B}\mathbf{w}^* - \frac{1}{\gamma_0}|\mathbf{c}\mathbf{w}^T|^2+1\right) \\ &\quad +\lambda_2\left(\mathbf{w}\mathbf{D}\mathbf{w}^* - \frac{1}{\gamma_1}|\mathbf{e}\mathbf{w}^T|^2+1\right), \end{aligned}$$

where $\mathbf{w} = [w_1\ \dots\ w_K]$, $\mathbf{A}$ is given at the top of the page,

$$\mathbf{B} = \begin{bmatrix} |g_1|^2 & 0 & \dots & 0 \\ 0 & |g_2|^2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \dots & 0 & |g_K|^2 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} |h_1^r|^2 & 0 & \dots & 0 \\ 0 & |h_2^r|^2 & \dots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & \dots & 0 & |h_K^r|^2 \end{bmatrix},$$

$\mathbf{c} = \begin{bmatrix} g_1 h_1 & \dots & g_K h_K \end{bmatrix}$, and $\mathbf{e} = \begin{bmatrix} h_1^r g_1^r & \dots & h_K^r g_K^r \end{bmatrix}$.

Differentiating the Lagrangian yields

$$\left(\mathbf{A}+\lambda_1\mathbf{B}+\lambda_2\mathbf{D}-\frac{\lambda_1}{\gamma_0}\mathbf{c}^*\mathbf{c}+\frac{\lambda_2}{\gamma_1}\mathbf{e}^*\mathbf{e}\right)\mathbf{w} = 0.$$

The optimal $\mathbf{w}$ is found by solving for λ_1 and λ_2 using the constraints [3].

Therefore, by recasting our original problem of obtaining the boundary points of $\mathcal{R}(P, P_R)$ to the power minimization problem with SINR constraints, we have shown that the optimal solution can be found in an efficient way. In Section V, we plot the achievable rate region of the optimal AF strategy and compare it with the cut-set upper bound derived in [21].

Recall that we only considered a two-way relay channel, where each relay had a single antenna, $N=1$. Extension to $N>1$, is straightforward by replacing $g_k w_k h_k$ by $\mathbf{g}_k\mathbf{W}_k\mathbf{h}_k$, $g_k w_k$ by $\mathbf{g}_k\mathbf{W}_k$, $h_k^r w_k g_k$ by $\mathbf{h}_k^r\mathbf{W}_k\mathbf{g}_k$ and $h_k^r w_k$ by $\mathbf{h}_k^r\mathbf{W}_k$, which are scalars as before, and the optimal solution to $\mathbf{W}_k$'s can be found using the iterative power minimization algorithm (5).

V. Simulations

In Figs. 2 and 3, we plot the achievable rate region of the optimal AF strategy, and compare it with the lower bound obtained using dual channel matching [21], and the cut-set upper bound [21], for $K=2$ and $K=4$, with $M=1$, $N=1$ and $P=P_R=10dB$ with sum rate constraint across relays. With the dual channel matching strategy [21], relay k multiplies $\sqrt{\beta_k}\left(\mathbf{G}_k^*\mathbf{H}_k^*+\mathbf{H}_k^{r*}\mathbf{G}_k^{r*}\right)$ to the received signal and forwards it to T_1 and T_2, where β_k is the normalization constant to satisfy the power constraint. Dual channel matching tries to match both the channels which the data streams from T_1 to T_2 and T_2 to T_1 experience at each relay node. Note that the achievable rate region of the optimal AF region is symmetric, as expected, because of the symmetry in parameters of communication in both directions in a two-way relay channel. From Figs. 2 and 3, it is clear that the gap between the achievable rate region of the optimal AF strategy and the upper bound is quite small. Thus the derived iterative algorithm presents a good solution for achieving near optimal rates in a two-way relay channel. Also notice that the difference between the upper and lower bound is less than the 3 bit bound of [9].

VI. Conclusion

In this paper we addressed the problem of finding optimal relay beamformers to maximize the achievable rate region of the two-way relay channel with multiple relays, when each relay uses AF. The use of AF strategy is motivated by the fact

[2] An immediate consequence of this property is that the optimal solution does not change if all g_k's are scaled by $e^{j\phi_1}$, or all h_k^r's are scaled by $e^{j\phi_2}$.

[3] Clearly, the optimal $\mathbf{w}$ lies in the null space of some matrix that is a function of $\mathbf{A}, \mathbf{B}, \mathbf{c}, \mathbf{e}$, and $\mathbf{D}$ and hence not unique.

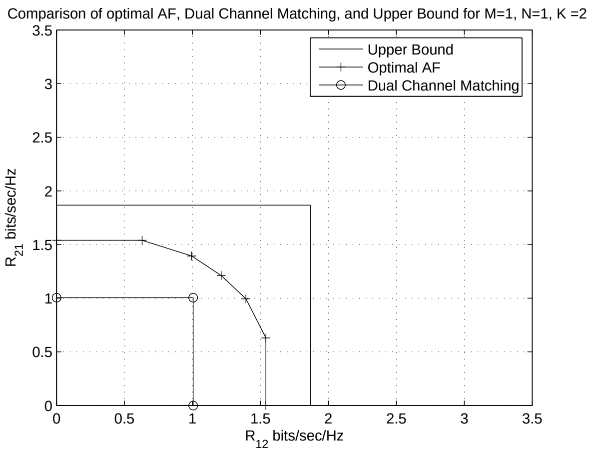

Fig. 2. Comparison of upper and lower bound of the capacity region of the two-way relay channel with $K = 2, M = 1, N = 1$, $P = P_R = 10dB$

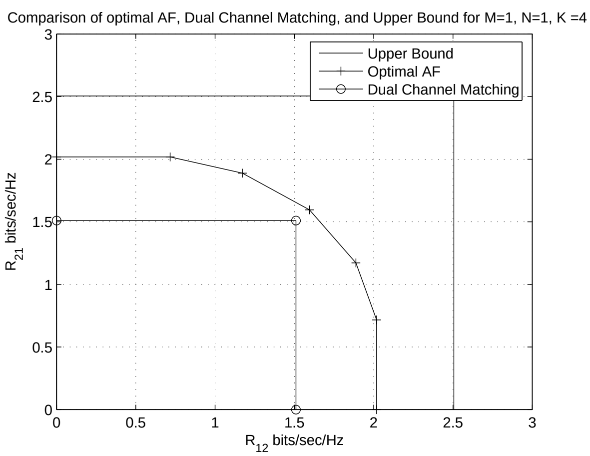

Fig. 3. Comparison of upper and lower bound of the capacity region of the two-way relay channel with $K = 4, M = 1, N = 1$, $P = P_R = 10dB$

that all the other known relay strategies such as DF, partial DF and CF, do not work well in the presence of multiple relays, and moreover, AF is quite simple to implement. For the case when both the terminals T_1 and T_2 have a single antenna and each relay has one or more antennas, we found an iterative algorithm to compute the optimal relay beamformers. The algorithm is equivalent to solving a power minimization problem subject to SINR constraints at each step. The power minimization problem at each step is non-convex, however, for which it is sufficient to satisfy the KKT conditions to obtain the optimal solution.

VII. Acknowledgments

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