Cascaded Orthogonal Space-Time Block Codes for Wireless Multi-Hop Relay Networks

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Abstract

Distributed space-time block coding is a diversity technique to mitigate the effects of fading in multi-hop wireless networks, where multiple relay stages are used by a source to communicate with its destination. This paper proposes a new distributed space-time block code called the cascaded orthogonal space-time block code (COSTBC) for the case where the source and destination are equipped with multiple antennas and each relay stage has one or more single antenna relays. Each relay stage is assumed to have receive channel state information (CSI) for all the channels from the source to itself, while the destination is assumed to have receive CSI for all the channels. To construct the COSTBC, multiple orthogonal space-time block codes are used in cascade by the source and each relay stage. In the COSTBC, each relay stage separates the constellation symbols of the orthogonal space-time block code sent by the preceding relay stage using its CSI, and then transmits another orthogonal space-time block code to the next relay stage. COSTBCs are shown to achieve the maximum diversity gain

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in a multi-hop wireless network with flat Rayleigh fading channels. Several explicit constructions of COSTBCs are also provided. It is also shown that COSTBCs require minimum decoding complexity thanks to the connection to orthogonal space-time block codes.

I. INTRODUCTION

Distributed space-time block coding (DSTBC) is a technique to improve reliability in a relay assisted communication, where one or more relays help the source to communicate with its destination. In DSTBCs, relay antennas are used together with the source antennas in a distributed manner to transmit a space time block code (STBC) [1] to the destination. By introducing redundancy in space and time, DSTBCs increase the reliability of the communication by increasing the diversity gain, defined as the negative of the exponent of the signal-to-noise ratio (SNR) in the pairwise error probability expression at high SNR [1].

Relay assisted communication is required in the deployment of large wireless networks, such as an ad-hoc network or a sensor network, where most often the destination is out of the source's communication range and multiple relays/hops are required for the source signal to reach the destination. Relay assisted communication is also used in a cellular wireless networks to improve the performance of cell edge users, and has been incorporated in wireless standards such as IEEE 802.16j and IEEE 802.16j and IEEE 802.11s. Consequently, there is a need for construction of DSTBCs that can achieve maximum diversity gain in the presence of two or more hops between source and destination. Unfortunately, most prior DSTBC designs [2]–[13] are restricted to a two-hop network. Moreover, the decoding complexity of known DSTBC designs [2]–[12] is quite high and none of them allow simple decoding like the Alamouti code [14], except for [13]. By exploiting extended Clifford algebra properties, the DSTBC construction [13] has lower decoding complexity than other maximum diversity gain DSTBC constructions [2]–[12], however, it still requires decoding complexity more than that of Alamouti code decoding.

In this paper we design maximum diversity gain achieving DSTBCs for multi-hop wireless networks that require minimum decoding complexity. Construction of DSTBCs with minimum decoding complexity is of importance from a practical implementation point of view as highlighted by the fact that the Alamouti code is the most practically used code not only because it achieves maximum diversity gain but also because it requires minimum decoding complexity. We assume that the source and the destination terminals have multiple antennas while the relays in each stage have one or more antennas. We also assume that each relay and the destination has perfect receive channel state information (CSI). The proposed DSTBCs can be used in both the full-duplex and half-duplex relay functionality, (for the half-duplex case the rate of transmission is halved).

In the proposed DSTCBC, called the cascaded orthogonal space-time block code (COSTBC), an orthogonal space-time code (OSTBC) [15] is used by the source and each relay stage to communicate with its adjacent relay stage. OSTBCs are considered because of their single symbol decodable property [14], [15], i.e. each constellation symbol of the OSTBC can be separated at the receiver with independent noise terms. With COSTBCs, in the first time slot the source transmits an OSTBC to the first relay stage. Using the single symbol decodable property of the OSTBC, each relay of the first relay stage separates the different OSTBC constellation symbols from the received signal and transmits a codeword vector in the next time slot, such that the matrix obtained by stacking all the codeword vectors transmitted by the different relays of the first relay stage is an OSTBC. These operations are repeated by subsequent relay stages [1].

We first prove that the COSTBCs have single symbol decodable property, i.e. each transmitted constellation symbol can be decoded independently of all the other constellation symbols, with no loss compared to joint detection. Then using the single symbol decodable property, we prove that COSTBCs achieve maximum diversity gain in a multiple hop wireless network.

During the preparation of this manuscript we came across three related papers on maximum

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[1] With COSTBCs, no signal is decoded at any of the relays, therefore COSTBC construction with single antenna relays is equivalent to COSTBC construction with multiple antenna relays. Thus without loss of generality in this paper we only consider COSTBC construction for single antenna relays.

diversity gain achieving DSTBC construction for multi-hop wireless networks [16]–[18] [2]. We briefly review them and compare with the proposed COSTBCs.

By extending the AF strategy with unitary transformation for two-hop wireless networks [3], maximum diversity gain achieving DSTBCs are constructed in [16] for single antenna multi-hop wireless network, where each node (the source, each relay and the destination) has single antenna. The focus of [17], [18] is on the construction of DSTBCs that can achieve the optimal diversity multiplexing (DM) tradeoff [19] in a multi-hop wireless network. In [17] a parallel AF strategy is proposed which divides the total number of paths from the source to the destination into non-overlapping groups and transmits an STBC with non-vanishing determinant property [20] through each group simultaneously. It is shown that this strategy achieves the maximum diversity gain and maximum multiplexing gain points of the optimal DM-tradeoff in a multi-hop wireless network. A strategy similar to delay diversity strategy [1] is proposed in [18] to achieve the DM-tradeoff for the half-duplex multi-hop wireless network where both the source and the destination are equipped with single antenna.

Compared to [16], COSTBC is a more general strategy and easy to implement, since it achieves maximum diversity gain even in the presence of multiple antennas at the source and relays, with minimum decoding complexity. In comparison to the strategies of [17], [18], COSTBCs only achieves the maximum diversity gain and not the maximum multiplexing gain. Due to the use of OSTBCs, however, the decoding complexity of COSTBC is significantly less than the strategies of [17], [18] and makes COSTBCs amenable for practical implementation in comparison to [17], [18] where STBCs with high decoding complexity are used. Thus, COSTBCs are well suited for relay assisted communication where relays are used to improve the cell coverage, by improving reliability of the users at the cell edge, while requiring minimum decoding complexity.

Notation: Let $\mathbf{A}$ denote a matrix, $\mathbf{a}$ a vector and a_i the i^{th} element of $\mathbf{a}$. The determinant and

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[2] A part of this paper has been presented at ITA San Diego, Jan. 2008 and ISIT 2008, Toronto July 2008. Due to space limitation, the ISIT 2008 paper contains only the results of this paper without any proofs. In the present paper, we present detailed proofs of the results, together with explicit code construction, and some simulation results

trace of matrix $\mathbf{A}$ are denoted by $\det(\mathbf{A})$ and $tr(\mathbf{A})$. The field of real and complex numbers is denoted by $\mathbb{R}$ and $\mathbb{C}$, respectively. The space of $M \times N$ matrices with complex entries is denoted by $\mathbb{C}^{M\times N}$. The Euclidean norm of a vector $\mathbf{a}$ is denoted by $|\mathbf{a}|$. An $m \times m$ identity matrix is denoted by $\mathbf{I}_m$ and $\mathbf{0}_m$ is as an all zero $m \times m$ matrix. The superscripts $^T, ^*, ^\dagger$ represent the transpose, transpose conjugate and element wise conjugate. The expectation of function $f(x)$ with respect to x is denoted by $\mathbb{E}f(x)$. A circularly symmetric complex Gaussian random variable x with zero mean and variance σ^2 is denoted as $x \sim \mathcal{CN}(0, \sigma)$. We use the symbol $\doteq$ to represent exponential equality i.e., let $f(x)$ be a function of x, then $f(x) \doteq x^a$ if $\lim_{x\to\infty} \frac{\log(f(x))}{\log x} = a$ and similarly $\dot{\leq}$ and $\dot{\geq}$ denote the exponential less than or equal to and greater than or equal to relation, respectively. To define a variable we use the symbol :=.

II. SYSTEM MODEL

Consider a multi-hop wireless network where a source terminal with M_0 antennas wants to communicate with a destination terminal with M_N antennas via $N-1$ relay stages as shown in Fig. 1. Each relay in any relay stage has a single antenna; M_n denotes the number of relays in the n^{th} relay stage. It is assumed that the relays do not generate their own data and only operate in half-duplex mode. No assumption is made on the half-duplex or full-duplex functionality of the relays. In the half-duplex case the rate of transmission gets halved. Similar to the model considered in [18], we assume that any relay of relay stage n can only receive the signal from any relay of relay stage $n-1$, i.e. we consider a directed multi-hop wireless network. This assumption is valid for the case when successive relay stages appear in increasing order of distance from the source towards the destination and any two relay nodes are chosen to lie in adjacent relay stages if they have sufficiently good SNR between them. In any practical setting there will be interference received at any relay node of stage n because of the signals transmitted from relay nodes of relay stage $0, \ldots, n-2$ and $n+2, \ldots, N-1$. Due to relatively large distances between non adjacent relay stages, however, this interference is quite small and we account for that in the additive noise term. Throughout this paper we refer to this multi-hop wireless network with $N-1$ relay stages as an N-hop network.

As shown in Fig. 1, the channel between the source and the i^{th} relay of the first stage of relays is denoted by $\mathbf{h}_i = [h_{1i}\ h_{2i}\ \dots\ h_{M_0 i}]^T,\ i = 1, 2, \dots, M_1$, between the j^{th} relay of relay stage s and the k^{th} relay of relay stage $s+1$ by $f^s_{jk},\ s = 0, 1, \dots, N-2,\ j = 1, 2, \dots, M_s,\ k = 1, 2, \dots, M_{s+1}$ and the channel between the relay stage $N-1$ and the ℓ^{th} antenna of the destination by $\mathbf{g}_\ell = [g_{1\ell}\, g_{2\ell}\ \dots\ g_{M_{N-1}\ell}]^T,\ \ell = 1, 2, \dots, M_N$. We assume that $\mathbf{h}_i \in \mathbb{C}^{M_0 \times 1}$, $f^s_{jk} \in \mathbb{C}^{1\times 1}$, $\mathbf{g}_l \in \mathbb{C}^{M_{N-1}\times 1}$ with independent and identically distributed (i.i.d.) $\mathcal{CN}(0,1)$ entries for all i, j, k, ℓ, s. We assume that the m^{th} relay of n^{th} stage knows $\mathbf{h}_i, f^s_{jk},\ \forall\ i,\ j,\ k, s = 1, 2, \dots, n-2,\ f^{n-1}_{jm}\ \forall j$ and the destination knows $\mathbf{h}_i, f^s_{jk}, \mathbf{g}_l,\ \forall\ i, j, k, l, s$. We further assume that all these channels are frequency flat and block fading, where the channel coefficients remain constant in a block of time duration T_c and change independently from block to block. We assume that the T_c is at least $\max\{M_0, M_1, \dots, M_{N-1}\}$.

A. Problem Formulation

Definition 1: (STBC) [21] A rate-L/T $T \times N_t$ **design $\mathbf{D}$** is a $T \times N_t$ matrix with entries that are complex linear combinations of L complex variables $s_1, s_2, \dots, s_L$ and their complex conjugates. A rate-L/T $T \times N_t$ STBC $\mathbf{S}$ is a set of $T \times N_t$ matrices that are obtained by allowing the L variables $s_1, s_2, \dots, s_L$ of the rate-L/T $T \times N_t$ design $\mathbf{D}$ to take values from a finite subset $\mathbb{C}^f$ of the complex field $\mathbb{C}$. The cardinality of $\mathbf{S} = |\mathbb{C}^f|^L$, where $|\mathbb{C}^f|$ is the cardinality of $\mathcal{C}$. We refer to $s_1, s_2, \dots, s_L$ as the constituent symbols of the STBC.

Definition 2: A DSTBC $\mathcal{C}$ for a N-hop network is a collection of codes $\{\mathbf{S}_0, \mathbf{S}_1, \dots, \mathbf{S}_{N-1}\}$, where $\mathbf{S}_0$ is the STBC transmitted by the source and $\mathbf{S}_n = [\mathbf{f}^1_n(\mathbf{S}_{n-1}) \dots \mathbf{f}^{M_n}_n(\mathbf{S}_{n-1})]$ is the STBC transmitted by relay stage n, where $\mathbf{f}^j_n(\mathbf{S}_{n-1})$ is the vector transmitted by the j^{th} relay of stage n which is a function of $\mathbf{S}_{n-1},\ j = 0, \dots, M_n, n = 1, \dots, N-1$. An example of a DSTBC is illustrated in Fig. 2.

Definition 3: The diversity gain [1], [3] of a DSTBC $\mathcal{C}$ is defined as

$$d_{\mathcal{C}} = -\lim_{E \to \infty} \frac{\log P_e(E)}{\log E},$$

$P_e(E)$ is the pairwise error probability (PEP) using coding strategy $\mathcal{C}$, and E is the sum of the transmit power used by each node in the network.

The problem we consider in this paper is to design DSTBCs that achieve the maximum diversity gain in a N-hop network. To identify the limits on the maximum possible diversity gain in a N-hop network, an upper bound on the diversity gain achievable with any DSTBC is presented next.

Theorem 1: The diversity gain $d_{\mathcal{C}}$ of DSTBC $\mathcal{C}$ for an N-hop network is upper bounded by $\min\{M_n M_{n+1}\} \quad n = 0, 1, \ldots, N-1$.

Proof: See Proposition 2.1 [17]. ■

Theorem 1 implies that the maximum diversity gain achievable in a N-hop network is equal to the minimum of the maximum diversity gain achievable between any two relay stages, when all the relays in each relay stage are allowed to collaborate. In the next section we propose COSTBCs that are shown to achieve this upper bound on the diversity gain.

III. CASCADED ORTHOGONAL SPACE-TIME CODE

In this section we introduce the COSTBC design for a N-hop network. Before introducing COSTBCs, we need the following definitions.

Definition 4: With $T \geq N_t$, a rate L/T $T \times N_t$ STBC $\mathbf{S}$ is called full-rank or fully-diverse or is said to achieve maximum diversity gain if the difference of any two matrices $\mathbf{M}_1, \mathbf{M}_2 \in \mathbf{S}$ is full-rank, $\min_{\mathbf{M}_1 \neq \mathbf{M}_2,\ \mathbf{M}_1, \mathbf{M}_2 \in \mathbf{S}} \text{rank}(\mathbf{M}_1 - \mathbf{M}_2) = N_t$.

Definition 5: A rate-L/K $K \times K$ STBC $\mathbf{S}$ is called an orthogonal space-time block code (OSTBC) if the design $\mathbf{D}$ from which it is derived is orthogonal i.e. $\mathbf{D}\mathbf{D}^* = (|s_1|^2 + \ldots + |s_L|^2)\mathbf{I}_K$.

Definition 6: Let $\mathbf{S}$ be a rate-L/K $K \times K$ STBC. Then, using CSI, if each of the constituent symbols $s_i,\ i = 1, \ldots, L$ of $\mathbf{S}$ can be separated/decoded independently of s_j $\forall i \neq j$ $i, j = 1, \ldots, L$ with independent noise terms, then $\mathbf{S}$ is called a single symbol decodable STBC.

Remark 1: OSTBCs are single symbol decodable STBCs [15].

With these definitions we are now ready to describe COSTBC for a N-hop network.

COSTBC is a DSTBC where each $\mathbf{S}_n,\ n = 0, 1, \ldots, N-1$ is an OSTBC. Thus, with COSTBC the source transmits a rate-L/M_0 $M_0 \times M_0$ OSTBC $\mathbf{S}_0$ in time slot of duration M_0. How to construct OSTBCs $\mathbf{S}_n, n = 1, \ldots, N-1$ is detailed in the following. Let $\mathbf{S}_0$ be a rate-L/M_0 $M_0 \times M_0$ OSTBC transmitted by the source to all the relays of relay stage 1. Then the received signal $\mathbf{r}_k^1 \in \mathbb{C}^{M_0 \times 1}$ at relay k of relay stage 1 can be written as

$$\mathbf{r}_k^1 = \sqrt{E_0}\mathbf{S}_0\mathbf{h}_k + \mathbf{n}_k^1 \tag{1}$$

where $\mathbb{E}\ tr(\mathbf{S}_0^*\mathbf{S}_0) = M_0$ and E_0 is the power transmitted by the source at each time instant. The noise $\mathbf{n}_k^1$ is the $M_0 \times 1$ spatio-temporal white complex Gaussian noise independent across relays with $\mathbb{E}\mathbf{n}_k^1\mathbf{n}_k^{1*} = \mathbf{I}_{M_0}$. Since $\mathbf{S}_0$ is an OSTBC, using CSI, the received signal $\mathbf{r}_k^1$ can be transformed into $\tilde{\mathbf{r}}_k^1 \in \mathbb{C}^{L \times 1}$, where

$$\tilde{\mathbf{r}}_k^1 = \sqrt{E_0}\underbrace{\begin{bmatrix} \sum_{m=1}^{M_0}|h_{mk}|^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{m=1}^{M_0}|h_{mk}|^2 \end{bmatrix}}_{\mathsf{H}}\mathbf{s} + \tilde{\mathbf{n}}_k^1 \tag{2}$$

and $\mathbf{s} = [s_1, s_2,\ \ldots,\ s_L]^T$ is the vector of the constituent symbols of the OSTBC $\mathbf{S}_0$, H is an $L \times L$ matrix and $\tilde{\mathbf{n}}_k^1$ is an $L \times 1$ vector with entries that are uncorrelated and $\mathcal{CN}(0, M_0)$ distributed. Then we normalize $\tilde{r}_k^1$ by $\mathsf{H}^{-\frac{1}{2}}$ to obtain $\hat{r}_k^1$, where

$$\hat{\mathbf{r}}_k^1 := \mathsf{H}^{-\frac{1}{2}}\tilde{\mathbf{r}}_k^1 = \sqrt{E_0}\mathsf{H}^{\frac{1}{2}}\mathbf{s} + \underbrace{\mathsf{H}^{-\frac{1}{2}}\tilde{\mathbf{n}}_k^1}_{\hat{\mathbf{n}}_k^1}, \tag{3}$$

where $\hat{\mathbf{n}}_k^1$ is an $L \times 1$ vector with entries that are uncorrelated and $\mathcal{CN}(0, 1)$ distributed.

Then, in the second time slot of duration M_1, relay k of relay stage 1 transmits $\mathbf{t}_k^1$, constructed from the signal (3)

$$\mathbf{t}_k^1 = \sqrt{\frac{E_1M_1}{L\gamma}}\left(\mathbf{A}_k\hat{\mathbf{r}}_k^1 + \mathbf{B}_k\hat{\mathbf{r}}_k^{1\dagger}\right), \tag{4}$$

where $\gamma = \mathbb{E}\ \hat{\mathbf{r}}_k^{1*}\hat{\mathbf{r}}_k^1$ to ensure that the average power transmitted by each relay at any time instant is E_1, i.e. $\mathbb{E}\,(\mathbf{t}_k^1)^\dagger\,(\mathbf{t}_k^1) = E_1$ and $\mathbf{A}_k$, $\mathbf{B}_k$ are $M_1 \times L$ matrices such that

$$\mathbf{A}_k^*\mathbf{B}_k = -\mathbf{B}_k^*\mathbf{A}_k,\ \text{and}\ tr(\mathbf{A}_k^*(l)\mathbf{A}_k(l) + \mathbf{B}_k^*(l)\mathbf{B}_k(l)) = 1, \tag{5}$$

$\forall\ k = 1, 2 \ldots, M_1,\ l = 1, 2, \ldots L$, where $\mathbf{A}_k(l)$ and $\mathbf{B}_k(l)$ denote the l^{th} column of $\mathbf{A}_k$ and $\mathbf{B}_k$, respectively and $\mathbf{S}_1 := [\mathbf{A}_1\mathbf{s} + \mathbf{B}_1\mathbf{s}^\dagger \ldots \mathbf{A}_{M_1}\mathbf{s} + \mathbf{B}_{M_1}\mathbf{s}^\dagger]$ is an OSTBC.

Under these assumptions, the $M_1 \times 1$ received signal at the i^{th} relay of relay stage 2 is

$$\begin{aligned}\mathbf{y}_i &= \sum_{k=1}^{M_1} \mathbf{t}_k g_{ki} + \mathbf{z}_i \qquad (6)\\ &= \sqrt{\frac{E_0 E_1 M_1}{L\gamma}} \underbrace{[\mathbf{A}_1\mathbf{s} + \mathbf{B}_1\mathbf{s}^\dagger\ \mathbf{A}_2\mathbf{s} + \mathbf{B}_2\mathbf{s}^\dagger\ \ldots\ \mathbf{A}_{M_1}\mathbf{s} + \mathbf{B}_{M_1}\mathbf{s}^\dagger]}_{\mathbf{S}_1} \hat{\mathbf{H}}^{\frac{1}{2}}\mathbf{g}_i\\ &\quad + \sqrt{\frac{E_1 M_1}{L\gamma}} [\mathbf{A}_1\hat{\mathbf{n}}_1^1 + \mathbf{B}_1\hat{\mathbf{n}}_1^{1\dagger}\ \ \ldots\ \mathbf{A}_{M_1}\hat{\mathbf{n}}_{M_1}^1 + \mathbf{B}_{M_1}\hat{\mathbf{n}}_{M_1}^{1\dagger}]\mathbf{g}_i + \mathbf{z}_i\end{aligned}$$

for $i = 1, 2, \ldots M_2$, where $\mathbf{z}_i$ is the $M_1 \times 1$ spatio-temporal white complex Gaussian noise independent across M_2 receive antennas with i.i.d. $\mathcal{CN}(0, 1)$ entries and

$$\hat{\mathbf{H}}^{\frac{1}{2}} = \begin{bmatrix} \sqrt{\sum_{m=1}^{M_0} |h_{m1}|^2} & 0 & 0 & 0\\ 0 & \sqrt{\sum_{m=1}^{M_0} |h_{m2}|^2} & 0 & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & 0 & \sqrt{\sum_{m=1}^{M_0} |h_{mM_1}|^2} \end{bmatrix}.$$

Thus, an OSTBC $\mathbf{S}_1$ is transmitted by relay stage 1 to the relay stage 2 in a distributed manner. To construct the COSTBC, the strategy of transmitting an OSTBC from relay stage 1 is repeated at each relay stage, i.e. each relay of relay stage n transforms the received signal as in (3) for the OSTBC transmitted from the relay stage $n-1$ and transmits an OSTBC in time duration M_n using $A_k, B_k, k = 1, \ldots, M_n$ together with all the other relays in relay stage n to the relay stage $n+1$. The power used up at each relay of relay stage n is E_n such that $E_0 + \sum_{n=1}^{N-1} M_n E_n = E$, where E is the total power available in the network. In the N^{th} time slot of duration M_{N-1} the receiver receives an OSTBC from relay stage $N-1$.

The properties of the COSTBC are summarized in the next two Theorems.

Theorem 2: COSTBCs are single symbol decodable STBCs.

Theorem 2 is proved in Appendix I.

Theorem 3: COSTBCs achieve the diversity gain upper bound (Theorem 1) in a N-hop network.

Theorem 3 is proved in Appendix II.

Discussion: In this section we constructed COSTBCs by cascading OSTBCs at each relay stage. OSTBCs are cascaded at each relay stage by first separating each constellation symbol of the OSTBC transmitted from the preceding relay stage and then transmitting another OSTBC to the next relay stage. The proposed OSTBC cascading strategy is novel and different than other approaches that use Alamouti code or OSTBC in a distributed manner [12], [22]. In [22], for the case of two-hop network, with a single relay and single antenna at the source and the relay, the relay node decodes the received signal from the source when the mutual information is more than the rate of transmission and then transmits an Alamouti code together with the source to the destination. In [12], [22] for the case of two-hop network, with multiple relays and single antenna at the source and each relay, the relay nodes scale the received signals and transmit an OSTBC in the next time slot. The key difference between COSTBCs and [12], [22] is that in COSTBCs all constellation symbol are decoupled at each relay stage without any decoding, and transmitted using an OSTBC to the next relay stage, rather than just scaling by each relay [12], or decoding by the relay [22]. As a result, COSTBCs simplify the problem of construction of maximum diversity gain DSTBCs for a N-hop network to the problem of construction of N-OSTBCs for each relay stage, which is well known [15]. Moreover, COSTBCs can be constructed for any number of antennas at the source and relays, and for multiple hop networks.

We showed that by cascading OSTBCs, the single symbol decodable property of OSTBCs is preserved. Therefore, COSTBCs require minimum decoding complexity, which is quite critical for practical implementations. We also proved that COSTBCs maximum diversity gain in a N-hop network. To obtain this result we used the single symbol decodable property of COSTBCs. Using the single symbol decodable property, the destination decouples the different constellation symbols of the OSTBC transmitted by the source, where the channel gain of each of the constellation symbols is shown to have at least $\min_{n=0,1,\dots,N-1}\{M_nM_{n+1}\}$ channel coefficients. Therefore the diversity gain of COSTBC is equal to $\min_{n=0,1,\dots,N-1}\{M_nM_{n+1}\}$ which meets the upper bound Theorem 1.

IV. EXPLICIT CODE CONSTRUCTIONS

In this section, we explicitly construct COSTBCs that achieve maximum diversity gain in N-hop networks. We present examples of COSTBCs for $N = 2$, $M_0 = M_1 = 2$ using the Alamouti code [14], $N = 2$, $M_0 = M_1 = 4$ using the rate-$3/4$ 4 antenna OSTBC [15] and $N = 2$, $M_0 = M_1 = 4$ using the rate-$3/4$ 4 antenna OSTBC and the Alamouti code.

Example 1: (Cascaded Alamouti Code) We consider $N = 2$, $M_0 = M_1 = 2$ case and let $\mathbf{S}_0$ be the Alamouti code given by: $\mathbf{S}_{ala} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}$ where s_1 and s_2 are constituent symbols of the Alamouti code. The 2×1 received signal at relay m is

$$\begin{bmatrix} r_{1m} \\ r_{2m} \end{bmatrix} = \sqrt{E_0} \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_{1m} \\ h_{2m} \end{bmatrix} + \begin{bmatrix} n_{1m} \\ n_2 m \end{bmatrix}$$

for $m = 1, 2$. Transforming this in the usual way

$$\begin{bmatrix} r_{1m} \\ -r_{2m}^* \end{bmatrix} = \sqrt{E_0} \underbrace{\begin{bmatrix} h_{1m} & h_{2m} \\ -h_{2m}^* & h_{1m}^* \end{bmatrix}}_{\tilde{\mathbf{H}}_m} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_{1m} \\ -n_{2m}^* \end{bmatrix}$$

for $m = 1, 2$. We define $\tilde{h}_m := |h_{1m}|^2 + |h_{2m}|^2$, $\eta_{1m} := (n_{1m} h_{1m}^* + n_{2m}^* h_{2m})$, and $\eta_{2m} := (n_{1m} h_{2m}^* - n_{2m}^* h_{1m})$. Pre-multiplying by $\tilde{\mathbf{H}}_m^*$,

$$\begin{bmatrix} \hat{r}_{1m} & \hat{r}_{2m}^* \end{bmatrix}^T := \tilde{\mathbf{H}}_m^* \begin{bmatrix} \hat{r}_{1m} & \hat{r}_{2m}^* \end{bmatrix}^T = \sqrt{E_0} \begin{bmatrix} \tilde{h}_m s_1 & \tilde{h}_m s_2 \end{bmatrix}^T + \begin{bmatrix} \eta_{1m} & \eta_{2m} \end{bmatrix}^T$$

for $m = 1, 2$. Now using

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{B}_1 = \mathbf{0}_2, \quad \mathbf{A}_2 = \mathbf{0}_2, \mathbf{B}_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{7}$$

the STBC $\mathbf{S}_1$ formed by the two relays is equal to $\mathbf{S}_{ala}^T$ which is also an OSTBC as required. Note that $\mathbf{A}_i, \mathbf{B}_i$ $i = 1, 2$ satisfy the requirements of (5). We call this the *cascaded Alamouti code*.

Example 2: In this example we consider the case $N = 2$, $M_0 = 4$, $M_1 = 4$. We choose $\mathbf{S}_0$ to be the rate-$3/4$ OSTBC for 4 transmit antennas given by

$$\mathbf{S}_0 = \begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ s_3^* & 0 & -s_1^* & s_2 \\ 0 & s_3^* & -s_2^* & -s_1 \end{bmatrix} \tag{8}$$

and use

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

and

$$\mathbf{B}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{B}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{B}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

It is easy to verify that $tr(\mathbf{A}_i^*\mathbf{A}_i + \mathbf{B}_i^*\mathbf{B}_i) = 3$ and $\mathbf{A}_i^*\mathbf{B}_i = -\mathbf{B}_i^*\mathbf{A}_i, \quad i = 1, 2, 3, 4$ as required. Then the STBC $\mathbf{S}_1 = \mathbf{S}_0$ using these $\mathbf{A}_i, \mathbf{B}_i \ i = 1, 2, 3, 4$, which is a rate-$3/4$ OSTBC as described above.

In both the previous examples we constructed COSTBC for $N = 2$-hop case by repeatedly using the same OSTBC at both the source and the relay stage. Using a similar procedure, it is easy to see that when $M_i = M_j \ \forall \ i, j = 0, 1, \ldots, N-1, \ i \neq j$ we can construct COSTBCs by using particular OSTBC for M_0 antennas at the source and each relay stage, e.g. if $\mathcal{O}$ is an OSTBC for M_0 antennas, then by using $\mathbf{S}_n = \mathcal{O}, \ n = 0, 1, \ldots, N-1$ we obtain a maximum diversity gain achieving COSTBC. OSTBC constructions for different number of antennas can be found in [15]. In the next example we construct COSTBC for $M_0 = 4$ and $M_1 = 2$ by cascading the rate-$3/4$ 4 antenna OSTBC with the Alamouti code.

Example 3: Let $N = 2,\ M_0 = 4$ and $M_1 = 2$. We choose $\mathbf{S}_0$ to be the rate-$3/4$ 4 antenna OSTBC (8) and $\mathbf{S}_1$ to be the Alamouti code. The COSTBC is constructed as follows.

Let $\mathbf{S}_0$ (8) be the transmitted rate-$3/4$ 4 antenna OSTBC from the source. Then the received signal at relay node $m,\ m = 1, 2$ is $\mathbf{r}^m = \sqrt{E_0}\mathbf{S}_0 \left[\begin{array}{cccc} h_{1m} & h_{2m} & h_{3m} & h_{4m} \end{array}\right]^T + \left[\begin{array}{cccc} n_1^m & n_2^m & n_3^m & n_4^m \end{array}\right]^T$. Using CSI the received signal $\mathbf{r}^m$ can be transformed into $\hat{\mathbf{r}}^m$, where $\hat{\mathbf{r}}^m := \left[\begin{array}{ccc} \hat{r}_1^m & \hat{r}_2^m & \hat{r}_3^m \end{array}\right]^T = \sqrt{E_0} \left[\begin{array}{ccc} \hat{h}_m s_1 & \hat{h}_m s_2 & \hat{h}_m s_3 \end{array}\right]^T + \left[\begin{array}{ccc} \hat{n}_1^m & \hat{n}_2^m & \hat{n}_3^m \end{array}\right]^T$ and $\hat{h}_m = \sqrt{\sum_{i=1}^{M_0} |h_{im}|^2}$. Then in the next time slot, the relay $m,\ m = 1, 2$ transmits $\mathbf{A}_m \left[\begin{array}{cc} \hat{r}_1^m & \hat{r}_2^m \end{array}\right]^T + \mathbf{B}_m \left[\begin{array}{cc} \hat{r}_1^m & \hat{r}_2^m \end{array}\right]^{T\dagger}$ where $\mathbf{A}_m, \mathbf{B}_m$ are given in (7).

These operations are repeated at the source and each relay stage in subsequent time slots. In the next time slot, signal s_3 received in the previous time slot and s_1 received in the current time slot is transmitted from relay stage 1 to the destination. Clearly, the relay stage transmits an Alamouti code which is an OSTBC and hence leads to a COSTBC construction for $M_0 = 4,\ M_1 = 2$. Using a similar technique as illustrated in this example, COSTBCs can be constructed for different number of source antenna and relay node configurations by suitably adapting different OSTBCs.

V. SIMULATION RESULTS

In this section we provide some simulation results to illustrate the bit error rates (BER) of COSTBCs for 2 and 3-hop networks. In all the simulation plots, E denotes the total power used by all nodes in the network, i.e. $E_0 + \sum_{n=1}^{N-1} M_n E_n = E$ and the additive noise at each relay and the destination is complex Gaussian with zero mean and unit variance. By equal power allocation between the source and each relay stage we mean $E_0 = M_n E_n = \frac{E}{N},\ \forall n = 1, \ldots, N-1$.

In Fig. 3 we plot the bit error rates of a cascaded Alamouti code and the comparable DSTBC from [3] with 4 QAM modulation for $N = 2,\ M_0 = M_1 = 2$ and $M_2 = 1, 2, 3$ with equal power allocation between the source and all the relays. It is easy to see that both the cascaded Alamouti code and the DSTBC from [3] achieves the maximum diversity gain of the 2-hop network, however, COSTBCs require 1 dB less power than the DSTBCs from [3], to achieve the same BER. The improved BER performance of COSTBCs over DSTBCs from [3], is due to fact that with COSTBCs, each relay coherently combines the signal received from the previous relay stage before forwarding it to the next relay stage, while no such combining is done in [3].

Next we plot the BER curves for $N = 2$, $M_0 = M_1 = 4$, and $N = 2$, $M_0 = 4,\ M_1 = 2$ configurations in Figs. 4 and 5 with different M_2 and using equal power allocation between the source and the relay stage. For the $M_0 = M_1 = 4$ case we use the cascaded rate-$3/4$ 4 antenna OSTBC and for the $M_0 = 4,\ M_1 = 2$ case we use a rate-$3/4$ 4 antenna OSTBC at the source and the Alamouti code across both the relays as discussed in Section IV. From Figs. 4 and 5 it is clear that both these codes achieve maximum diversity gain for the respective network configurations.

Finally, in Fig. 6 we plot the bit error rates of a cascaded Alamouti code with $N = 3$-hop network where $M_0 = M_1 = M_2 = 2$ with $M_3 = 1, 2, 3$, and the cascaded Alamouti code is generated by repeated use of the Alamouti code by each relay stage with equal power allocation between the source and the relay stages. In this case also it is clear that the cascaded Alamouti code achieves the maximum diversity gain but there is a SNR loss compared to $N = 2$ case, because of the noise added by one extra relay stage.

From all the simulation plots, it is clear that COSTBCs require large transmit power to obtain reasonable BER's with multi-hop wireless networks. This is a common phenomenon across all the maximum diversity gain achieving DSTBC's for multi-hop wireless networks that use AF [3], [5], [9]. With AF, the noise received at each relay gets forwarded towards the destination and limits the received SNR at the destination, however, without using AF it is difficult to achieve maximum diversity gain in a multi-hop wireless network.

VI. CONCLUSION

In this paper we designed COSTBCs that achieve the maximum diversity gain in a multi-hop wireless network. We also showed that COSTBCs are single symbol decodable similar to OSTBCs and thus incur minimum decoding complexity, making them well suited for practical implementation. We then gave an explicit construction of COSTBCs for various numbers of source, destination, and relay antennas. The only restriction that COSTBCs impose is that the source and all the relay stages have to use an OSTBC. It is well known that high rate OSTBC do not exist, therefore the COSTBCs have rate limitations. For future work it will be interesting to see whether the OSTBC requirement can be relaxed without sacrificing the maximum diversity gain and minimum decoding complexity of the COSTBCs.

APPENDIX I

In this section we prove that COSTBCs have the single symbol decodable property. We first show this for 2-hop networks and then generalize it to N-hop networks where N is any arbitrary integer. Let $\mathbf{S}_0$ be the transmitted OSTBC from the source and $\mathbf{s} = [s_1, \ldots, s_L]^T$ be the vector of the constituent symbols of $\mathbf{S}_0$. Then from (3), using CSI, the received signal $\mathbf{r}_k^1$ at the k^{th} relay of relay stage 1 can be transformed into $\hat{\mathbf{r}}_k^1$ where $\hat{\mathbf{r}}_k^1 = \sqrt{E_0}\mathsf{H}^{\frac{1}{2}}\mathbf{s} + \hat{\mathbf{n}}_k^1$, H is defined in (2) and the entries of $\hat{\mathbf{n}}_k$ are independent and $\mathcal{CN}(0,1)$ distributed. For $N = 2$, from (6) the received signal at the j^{th} antenna of the destination can be written as $\mathbf{y}_j = [\mathbf{t}_1^1 \ \mathbf{t}_2^1 \ \ldots \ \mathbf{t}_{M_1}^1]\mathbf{g}_j + \mathbf{z}_j$ for $j = 1, 2, \ldots M_2$, where $\mathbf{t}_k^1$ is the transmitted vector from relay k (4) of relay stage 1. The received signal $\mathbf{y}_j$ can also be written as

$$\begin{aligned}\mathbf{y}_j \;&=\; \sqrt{\frac{E_0E_1M}{L\gamma}}\mathbf{S}_1 \left[\ \sqrt{\textstyle\sum_{m=1}^{M_0}|h_{m1}|^2}g_{1j} \ \ \sqrt{\textstyle\sum_{m=1}^{M_0}|h_{m2}|^2}g_{2j} \ \ \ldots \ \ \sqrt{\textstyle\sum_{m=1}^{M_0}|h_{mM_1}|^2}g_{M_1j} \ \right]^T \\ &\quad \underbrace{+\sqrt{\frac{E_1M_1}{L\gamma}}[\mathbf{A}_1\hat{n}_1 + \mathbf{B}_1\hat{n}_1^\dagger \ \mathbf{A}_2\hat{n}_2 + \mathbf{B}_2\hat{n}_2^\dagger \ \ldots \ \mathbf{A}_{M_1}\hat{n}_{M_1} + \mathbf{B}_{M_1}\hat{n}_{M_1}^\dagger]\mathbf{g}_j + \mathbf{z}_j}_{\mathbf{w}_j}\end{aligned}$$

where $\mathbf{S}_1 = [\mathbf{A}_1\mathbf{s} + \mathbf{B}_1\mathbf{s}^\dagger \ \mathbf{A}_2\mathbf{s} + \mathbf{B}_2\mathbf{s}^\dagger \ \ldots \ \mathbf{A}_{M_1}\mathbf{s} + \mathbf{B}_{M_1}\mathbf{s}^\dagger]$.

Since $\mathbf{S}_1$ is an OSTBC, invoking the single symbol decodable property of OSTBC (2) and using the fact that entries of $\mathbf{w}_j$ are independent, it follows that, using CSI, the received signal $\mathbf{y}_j$ can be transformed into $\hat{\mathbf{y}}_j$, where

$$\hat{\mathbf{y}}_j = \sqrt{\frac{E_0E_1M_1}{L\gamma}}\begin{pmatrix} \sum_{k=1}^{M_1}|g_{kj}|^2\left(\sum_{m=1}^{M_0}|h_{mk}|^2\right) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{k=1}^{M_1}|g_{kj}|^2\left(\sum_{m=1}^{M_0}|h_{mk}|\right)^2 \end{pmatrix}\mathbf{s} + \hat{\mathbf{w}}_j$$

and the entries of $\hat{\mathbf{w}}_j$ are independent. Thus, it is clear that all the constituent symbols $s_1, \ldots, s_L$ can be separated with independent noise terms and we conclude that COSTBCs have the single symbol decodable property for a 2-hop network. Now assume that the result is true for a k-hop network. Then by induction hypothesis, the received signal at the j^{th} antenna of destination of the k-hop network can be written as $\hat{\mathbf{r}}_j^k = p_{k-1}\mathsf{H}_{jk}^{\frac{1}{2}}\mathbf{s} + \hat{\mathbf{n}}_j^k$, where p_{k-1} is a function of $E_0, \ldots, E_{k-1}$, H_{jk} is a diagonal matrix with each entry equal to the channel gain and the entries of $\hat{\mathbf{n}}_j^k$ are independent and $\mathcal{CN}(0,1)$ distributed.

Now we extend the k-hop network to a $k+1$-hop network by assuming that the actual destination to be one more hop away and using the destination of the k-hop case as the k^{th} relay stage with M_k relays by separating the M_k antennas into M_k relays with single antenna each. With this extension, let the OSTBC transmitted by k^{th} relay stage be $\mathbf{S}_k$ using received signal $\mathbf{r}_j^k$, then the received signal at the i^{th} antenna of the destination of $k+1$-hop network is

$$\begin{aligned} \mathbf{r}_i^{k+1} &= p_k\mathbf{S}_k \begin{bmatrix} \mathsf{H}_{1k}^{\frac{1}{2}}(1)g_{1i} & \ldots & \mathsf{H}_{M_k k}^{\frac{1}{2}}(1)g_{M_1 i} \end{bmatrix}^T \\ &\quad + \underbrace{E_k[\mathbf{A}_1^k\hat{n}_1^k + \mathbf{B}_1^k\hat{n}_1^{k\dagger} \ \ldots \ \mathbf{A}_{M_1}^k\hat{n}_{M_k}^k + \mathbf{B}_{M_k}^k\hat{n}_{M_k}^{k\dagger}]\mathbf{g}_j + \mathbf{z}_i^{k+1}}_{\mathbf{w}_i^{k+1}}, \end{aligned}$$

where $\mathsf{H}_{jk}(1),\ j = 1, 2, \ldots, M_k$ is the $(1,1)$ element of H_{1k}, p_k is a function of $E_0, \ldots, E_k$, $\mathbf{S}_k = [\mathbf{A}_1^k\mathbf{s}+\mathbf{B}_1^k\mathbf{s}^\dagger\ \mathbf{A}_2^k\mathbf{s}+\mathbf{B}_2^k\mathbf{s}^\dagger\ \ldots\ \mathbf{A}_{M_k}^k\mathbf{s}+\mathbf{B}_{M_k}^k\mathbf{s}^\dagger]$ and $\mathbf{z}_i^{k+1}$ is $\mathcal{CN}(0,1)$ added at the destination.

Since $\mathbf{S}_k$ is an OSTBC, therefore, using CSI, the received signal at the destination of the $k+1$-hop network can be transformed to $\hat{\mathbf{r}}_i^{k+1}$, where

$$\hat{\mathbf{r}}_i^{k+1} = p_k \begin{pmatrix} \sum_{j=1}^{M_k}|g_{ji}|^2\mathsf{H}_k(1) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sum_{j=1}^{M_k}|g_{ji}|^2\mathsf{H}_{jk}(1) \end{pmatrix} \mathbf{s} + \hat{\mathbf{w}}_i^{k+1}$$

and the entries of $\hat{\mathbf{w}}_i^{k+1}$ are independent. Thus, we conclude that the COSTBCs have single symbol decodable property.

APPENDIX II

We prove Theorem 3 using induction. First we show that COSTBCs achieve maximum diversity gain for $N = 2$ and then extend the result for a k-hop network, where k is any arbitrary natural number.

The outage probability $P_{out}(R)$ is defined as $P_{out}(R) := P\left(I(\mathbf{s};\mathbf{r}) \leq R\right)$, where $\mathbf{s}$ is the input and $\mathbf{r}$ is the output of the channel and $I(\mathbf{s};\mathbf{r})$ is the mutual information between $\mathbf{s}$ and $\mathbf{r}$ [23]. Let $d_{out}(r)$ be the SNR exponent of P_{out} with rate of transmission R scaling as $r \log \mathsf{SNR}$, i.e. $\log P_{out}(r \log \mathsf{SNR}) \doteq \mathsf{SNR}^{-d_{out}(r)}$. Then, if $P_e(\mathsf{SNR}) \doteq \mathsf{SNR}^{-d(r)}$, then from [19], and the compound channel argument [18], $P_{out}(r \log \mathsf{SNR}) \doteq P_e(\mathsf{SNR}),\ d(r) \doteq d_{out}(r)$. Therefore to compute $d(r)$, it is sufficient to compute $d_{out}(r)$. In the following we compute $d_{out}(r)$ for the COSTBC with a 2-hop network.

For the 2-hop network, using the single symbol decodable property of COSTBCs (Appendix I), the received signal can be separated in terms of the individual constituent symbols of the OSTBC transmitted by the source. Therefore, the received signal can be written as

$$r_l = \sqrt{\theta E} \sum_{j=1}^{M_2} \sum_{k=1}^{M_1} |g_{kj}|^2 \left(\sum_{m=1}^{M_0} |h_{mk}|^2 \right) s_l + z_l \tag{9}$$

where θ is the normalization constant so as to ensure the total power constraint of E in the network, s_l is the $l^{th},\ \ l = 1, 2, \ldots, L$ symbol transmitted from the source and z_l is the additive white Gaussian noise (AWGN) with variance σ^2. Note that z_l depends on the channel coefficients, however, as shown in Theorem 1.2 [18], z_l is white in the scale of interest and without loss of generality z_l can be modeled as $\mathcal{CN}(0, \sigma^2)$ that is independent of channel coefficients in the outage analysis. Let $\mathsf{SNR} := \frac{\theta E}{\sigma^2}$, then

$$\begin{aligned} P_{out}(r \log \mathsf{SNR}) &= P\left(\log \left(1 + \mathsf{SNR} \sum_{j=1}^{M_2} \sum_{k=1}^{M_1} |g_{kj}|^2 \left(\sum_{m=1}^{M_0} |h_{mk}|^2 \right) \right) \leq r \log \mathsf{SNR} \right) \\ &\dot{\leq} P\left(\sum_{k=1}^{M_1} \sum_{j=1}^{\min\{M_0, M_2\}} |g_{kj}|^2 |h_{jk}|^2 \leq \mathsf{SNR}^{-(1-r)} \right) \\ &\leq P\left(\max_{\{j=1,\ldots,\min\{M_0,M_2\},\ k=1,\ldots,M_1\}} |g_{kj}|^2 |h_{jk}|^2 \leq \mathsf{SNR}^{-(1-r)} \right). \end{aligned}$$

Since $|g_{kj}|^2 |h_{jk}|^2$ are i.i.d. for $j = 1, \ldots, \min\{M_0, M_2\},\ k = 1, \ldots, M_1$ and the total number of terms are $\min\{M_0 M_1,\ M_1 M_2\}$,

$$P_{out}(r \log \mathsf{SNR}) \doteq P\left(|g_{11}|^2 |h_{11}|^2 \leq \mathsf{SNR}^{-(1-r)} \right)^{\min\{M_0 M_1,\ M_1 M_2\}}.$$

Note that $P\left(|g_{11}|^2|h_{11}|^2 \leq \mathsf{SNR}^{-(1-r)}\right)$ is the outage probability of a single input single output system which can be computed easily using [19] and is given by

$$P\left(|g_{11}|^2|h_{11}|^2 \leq \mathsf{SNR}^{-(1-r)}\right) \doteq \mathsf{SNR}^{-(1-r)},\ r \leq 1.$$

Thus, $P_{out}(r\log\mathsf{SNR}) \doteq \mathsf{SNR}^{-\min\{M_0M_1,\ M_1M_2\}(1-r)},\ r \leq 1$, and we have that $d_{out}(r) = \min\{M_0M_1,\ M_1M_2\}(1-r),\ r \leq 1$. Thus, the maximum diversity gain of the COSTBCs is $d_{out}(0) = \min\{M_0M_1,\ M_1M_2\}$ which equals the diversity gain upper bound (Theorem 1). Thus, we have shown that COSTBCs achieve maximum diversity gain in a 2-hop network. Next, using induction, we prove the result for any k-hop network.

Assume that the result is true for a k-hop network $(k \geq 2)$ and we will prove that it is true for a $k+1$-hop network. For a k-hop network using the single symbol decodable property of COSTBCs as shown in Appendix I, at the destination the received signal can be separated in terms of the individual constituent symbols of the OSTBC transmitted by the source. Thus the received signal can be written as

$$r_\ell = \sqrt{\theta E}\sum_{i=1}^{M_k} c_i s_\ell + z_\ell, \tag{10}$$

where θ is the normalization constant so as to ensure the total power constraint of E in the network, s_ℓ is the $\ell^{th},\ \ \ell = 1, 2, \ldots, L$ symbol transmitted from the source, c_i is the channel gain experienced by s_ℓ at the i^{th} antenna of the destination, and z_l is the additive white Gaussian noise (AWGN) with variance σ_k^2.

Now we extend the k-hop network to a $k+1$-hop network by assuming that the actual destination to be one more hop away and using the destination of the k-hop case as the k^{th} relay stage with M_k relays by separating the M_k antennas into M_k relays with single antenna each. Again using the single symbol decodable property of COSTBCs for the $k+1$-hop network, as shown in the Appendix I, the received signal at the destination can be separated in terms of individual constituent symbols of the OSTBC transmitted by the source, which is given by

$$y_\ell = \sqrt{\kappa E}\sum_{i=1}^{M_k} c_i \left(\sum_{j=1}^{M_{k+1}} |g_{ij}|^2\right) s_\ell + n_\ell, \quad \ell = 1, \ldots, L \tag{11}$$

where κ is a constant to ensure the power constraint of E in the $k+1$-hop network, g_{ij} is the channel between the i^{th} relay of relay stage k and the j^{th} antenna of the destination and n_l is the AWGN with variance σ_{k+1}^2. Let $y_\ell = \sum_{i=1}^{M_k} y_{\ell i}$, where

$$y_{\ell i} = \sqrt{\kappa E} q_i s_\ell + n_{\ell i} \tag{12}$$

for each $\ell = 1, \ldots, L$, where and $n_{\ell i} = n_\ell / M_k$. Recall from induction hypothesis that the diversity gain of COSTBCs with channel $c_i,\ \forall i$ (10) is $\alpha := \min\left\{\min\left\{M_n M_{n+1}\right\},\ M_{k-1}\right\},\quad n = 0, 1, \ldots, k-2$, by restricting the destination of the k-hop network to have only single antenna, and with channel $\sum_{i=1}^{M_k} c_i$ is $\min\left\{M_n M_{n+1}\right\},\ n = 0, 1, \ldots, k-1$, respectively. Thus, if the diversity gain of COSTBCs with channel q_i (12) is $\min\left\{\min\left\{M_n M_{n+1}\right\},\ M_{k-1}, M_{k+1}\right\}\ n = 0, 1, \ldots, k-2$, then, since $\sum_{j=1}^{M_{k+1}} |g_{ij}|^2$ are independent $\forall\ i$, it follows that the diversity gain of COSTBCs with channel $\sum_{i=1}^{M_k} q_i$ is $\min\left\{M_n M_{n+1}\right\},\ n = 0, 1, \ldots, k$. Next, we show that the diversity gain of COSTBCs with channel q_i is $\min\left\{\min\left\{M_n M_{n+1}\right\},\ M_{k-1}, M_{k+1}\right\},\quad n = 0, 1, \ldots, k-2$.

To compute the diversity gain of COSTBCs with channel q_i (12), we use the outage probability formulation [19] as follows. Let σ^2 be the variance of $n_{\ell i}$ (12), $\sigma^2 = \frac{\sigma_{k+1}^2}{M_k^2}$, and as before $\mathsf{SNR} := \frac{\kappa E}{\sigma^2}$, then the outage probability of (12) is

$$P_{out}(r \log \mathsf{SNR}) \;:=\; P\left(\log\left(1 + \mathsf{SNR} c_i \sum_{j=1}^{M_{k+1}} |g_{ij}|^2\right) \leq r \log \mathsf{SNR})\right).$$

$$\begin{aligned}
P_{out}(r \log \mathsf{SNR}) \;&\doteq\; P\left(c_i \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)}\right) \\
&=\; P\left(\sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)}\right) P\left(c_i \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)} | \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)}\right) \\
&\quad + P\left(\sum_{j=1}^{M_{k+1}} |g_{ij}|^2 > \mathsf{SNR}^{-(1-r)}\right) P\left(c_i \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)} | \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 > \mathsf{SNR}^{-(1-r)}\right) \\
&\leq\; P\left(\sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)}\right) + P\left(c_i \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 \leq \mathsf{SNR}^{-(1-r)} | \sum_{j=1}^{M_{k+1}} |g_{ij}|^2 > \mathsf{SNR}^{-(1-r)}\right).
\end{aligned}$$

Let $Z := \sum_{j=1}^{M_{k+1}} |g_{ij}|^2$. Then

$$P_{out}(r \log \mathsf{SNR}) \;\dot{\leq}\; P\left(Z \leq \mathsf{SNR}^{-(1-r)}\right) + \int_{\mathsf{SNR}^{-(1-r)}}^{\infty} \int_0^{\mathsf{SNR}^{-(1-r)}/z} f_{c_i}(y) dy f_Z(z) dz.$$

By induction hypothesis, the diversity gain of COSTBCs with c_i is α, i.e.,

$$P\left(c_i \leq \frac{\mathsf{SNR}^{-(1-r)}}{z}\right) = \int_0^{\mathsf{SNR}^{-(1-r)}/z} f_{c_i}(y)dy \leq k_4\left(\frac{\mathsf{SNR}^{-(1-r)}}{z}\right)^{\alpha}$$

where k_4 is a constant. Thus,

$$P_{out}(r\log\mathsf{SNR}) \;\leq\; P\left(Z\leq \mathsf{SNR}^{-(1-r)}\right) + \int_{\mathsf{SNR}^{-(1-r)}}^{\infty} k_4\mathsf{SNR}^{-\alpha(1-r)}\left(\frac{1}{z}\right)^{\alpha} f_Z(z)dz. \quad (13)$$

Since Z is a gamma distributed random variable with PDF $\frac{e^{-z}z^{M_{k+1}-1}}{M_{k+1}-1!}$, the first term in $P_{out}(r\log\mathsf{SNR})$ expression can be found in [19] and is given by $P\left(Z\leq\mathsf{SNR}^{-(1-r)}\right)\doteq\mathsf{SNR}^{-M_{k+1}(1-r)}$. Now we are left with computing the second term which can be done as follows.

$$\begin{aligned}\int_{\mathsf{SNR}^{-(1-r)}}^{\infty} k_4\mathsf{SNR}^{-\alpha(1-r)}\left(\frac{1}{z}\right)^{\alpha} f_Z(z)dz &\doteq k_4\mathsf{SNR}^{-\alpha(1-r)}\int_{\mathsf{SNR}^{-(1-r)}}^{\infty} z^{-\alpha}\frac{e^{-z}z^{M_{k+1}-1}}{M_{k+1}-1!}dz\\ &\doteq \mathsf{SNR}^{-\alpha(1-r)}.\end{aligned}$$

Thus, from (13) it follows that

$$P_{out}(r\log\mathsf{SNR})\doteq\mathsf{SNR}^{-M_{k+1}(1-r)}+\mathsf{SNR}^{-\alpha(1-r)}\doteq\mathsf{SNR}^{-\min\{M_{k+1},\alpha\}(1-r)}.$$

Using the definition of diversity gain, it follows that the diversity gain of COSTBCs with channel q_i is equal to $\min\{\alpha, M_{k+1}\}$, which implies that the diversity gain of COSTBCs with channel received signal model (11) is $\min\{\alpha M_k, M_kM_{k+1}\}$. Note that the upper bound on the diversity gain (Theorem 1) is also $\min\{\alpha M_k, M_kM_{k+1}\}$ and we conclude that the COSTBCs achieve the maximum diversity gain in a N-hop network.

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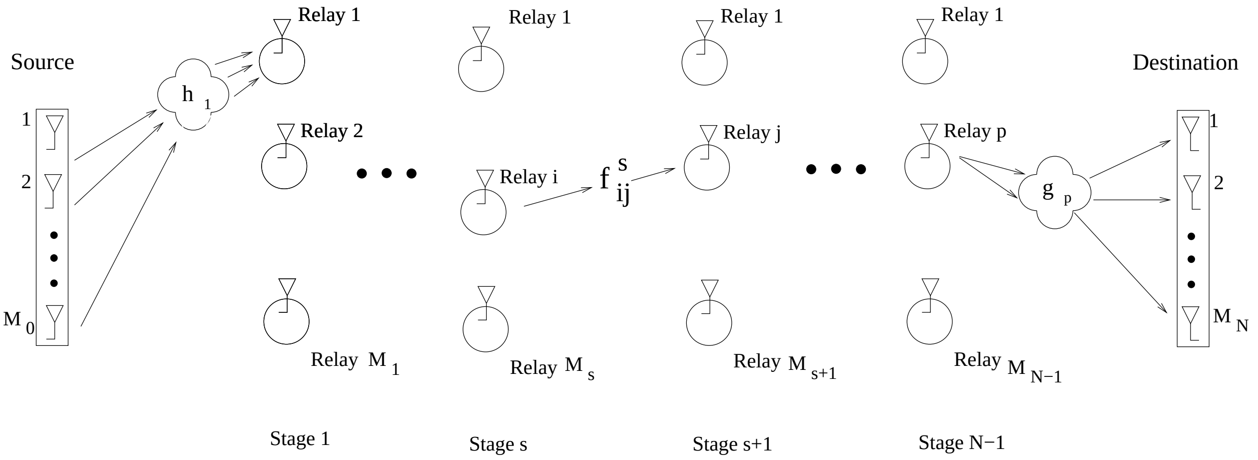

Fig. 1. System Block Diagram of a N-hop Wireless Network

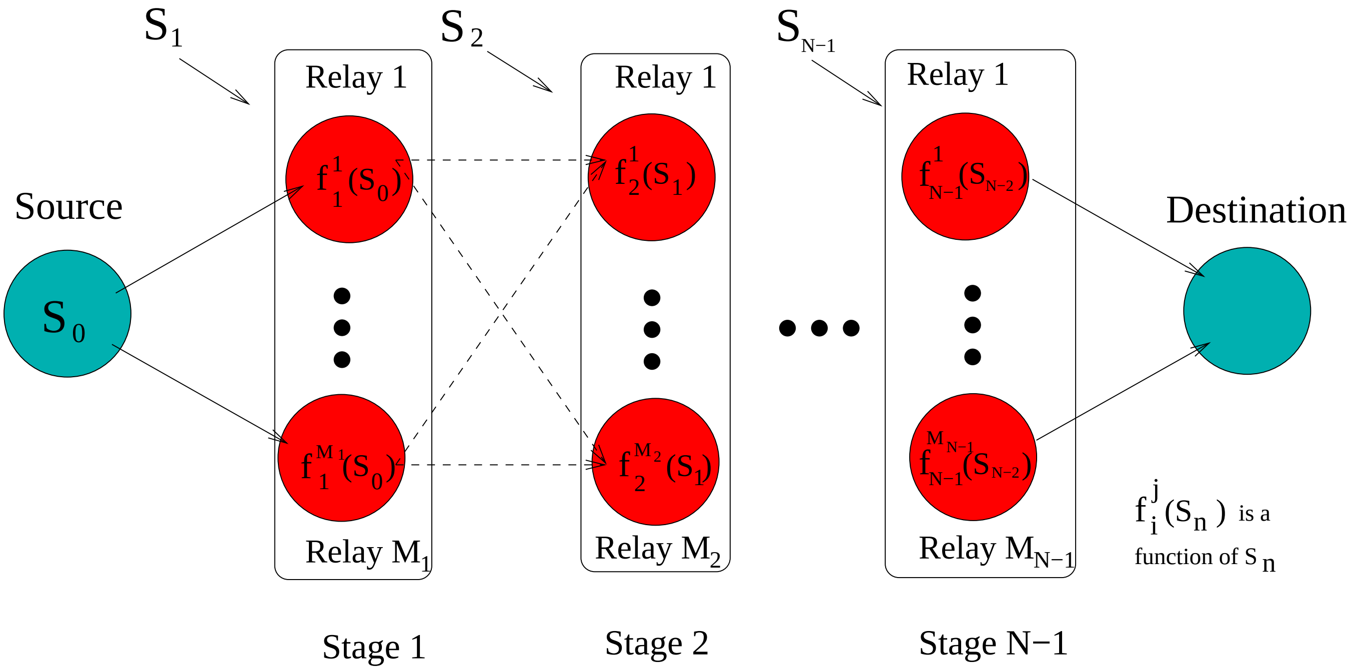

Fig. 2. An Illustration Of The DSTBC Design Problem

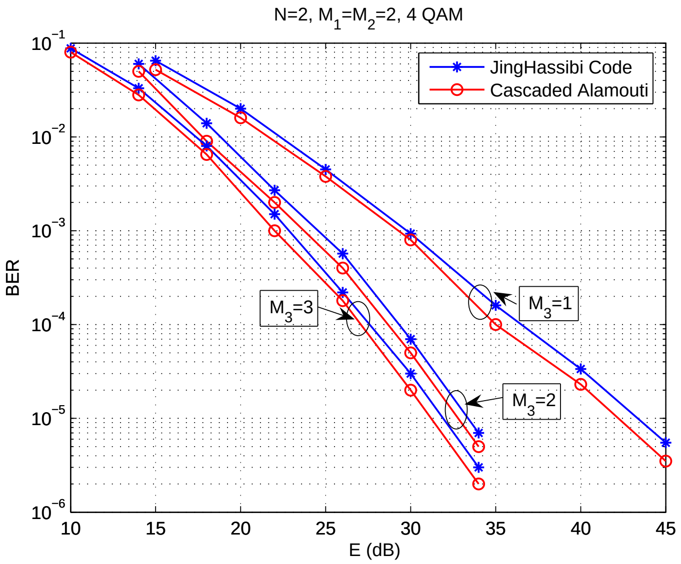

Fig. 3. BER comparison of Cascaded Alamouti code with JingHassibi code for $N = 2$-hop network

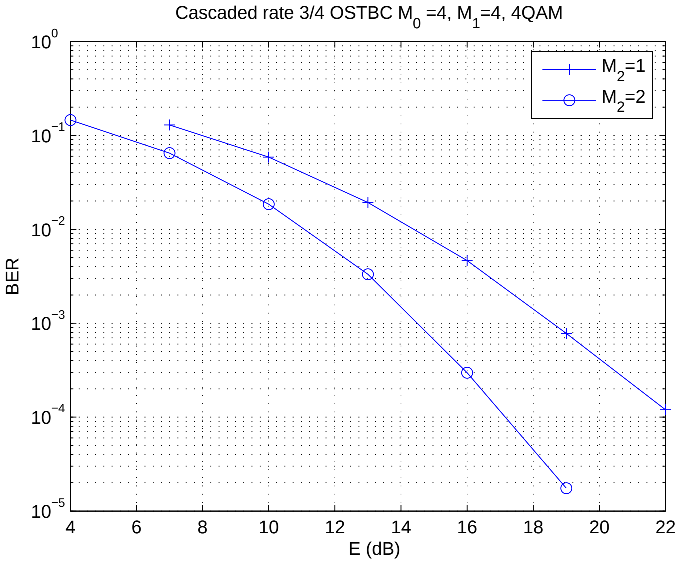

Fig. 4. Cascaded rate 3/4 4 antenna OSTBC for $M_0 = M_1 = 4$

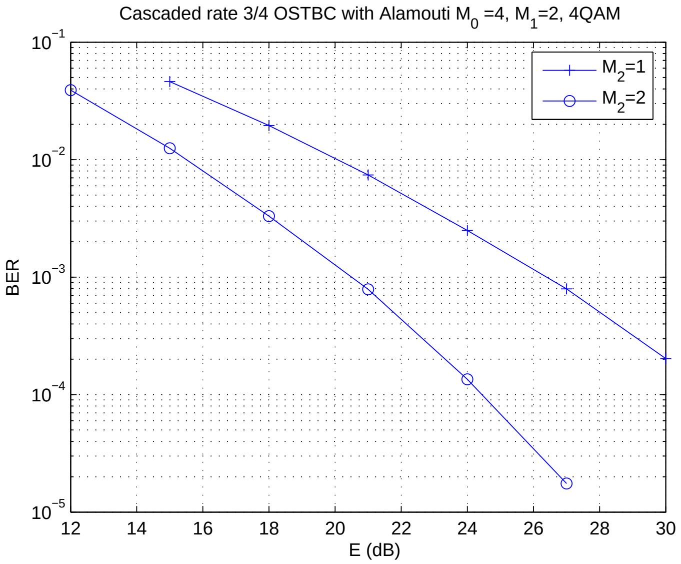

Fig. 5. Cascaded rate 3/4 4 antenna OSTBC with Alamouti Code for $M_0 = M_1 = 4$

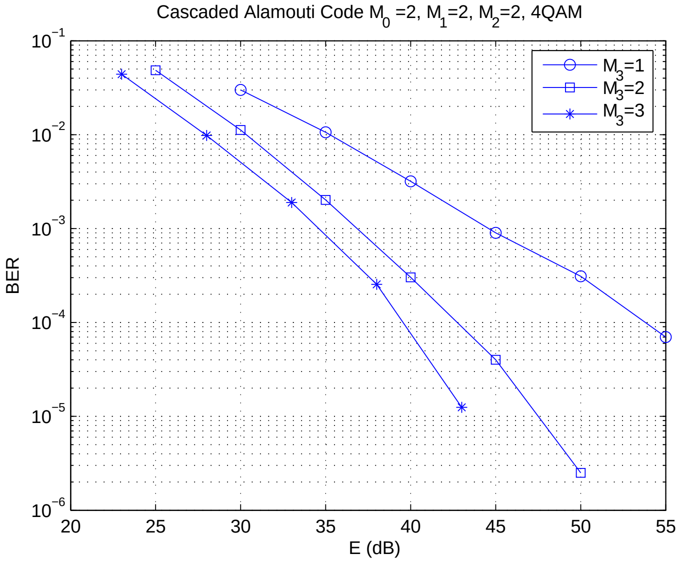

Fig. 6. Cascaded Alamouti Code for $N = 3$-hop network